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Assessing Students' Understanding of Tables and Graphs: Implications for Teaching and Research

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ABSTRACT

The importance of statistical literacy in everyday life and work place have led to calls for an increased attention to data display in the mathematics curriculum. More specifically, students should be able to ask worry questions and make judgements about validity of claims made in graphical representations. Different research studies investigating how well people interpret tables and graphs have found a wide range of responses, from no consideration of contexts to over consideration of contexts. This paper provides a meta-analysis of studies that have investigated peoples' thinking in this area. It aims to synthesize the results of these studies, and provide a conceptual framework for assessing information given in data representations. The paper concludes with implications for teaching and further research.

Keywords: Mathematics, meta-analysis

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INTRODUCTION

"For many people the first word that comes to mind when they think about statistical charts is lie. No doubt some graphics do distort the underlying data, making it hard for the viewer to learn the truth. But data graphs are no different from words in this regard, for any means of communication can be used to deceive. There is no reason to believe that graphs are especially vulnerable to exploitation by liars; in fact, most of us have pretty good graphical lie detectors that help us see right through frauds" (Tufté, 1983, p. 53)

Tufté's statement of graphs as a means of communication with potential to mislead is an important issue to consider in statistics curriculum today (Tishkovskaya, and Lancaster, 2012; Watson, 2006). This is especially true when advances in technology and communication have increased the widespread use of graphs of many kinds delivered through everyday media and advertising (Kemp and Kissane, 2010). People across the world are exposed to tables, graphs and statistical information in their everyday life and workplace. His view that most people have "pretty good graphical lie detectors" is not so straightforward as it seems and may be based on an assumption that graphs are transparent in communicating their meanings (Ainley, 2000). According to Best (2001) and Huff (1993), many graphs, especially in the media with its tendency toward sensational reporting may be designed to mislead, highlight, or hide a specific trend or difference. Hence, it cannot be ensured that graphic producers have followed the guidelines to guarantee graphical integrity (Tufté's lie detector), it is important that all citizens develop skills to understand graphs, be able to read them critically and ask questions.

The importance of statistics in everyday life and work place have led to calls for an increased attention to statistical literacy in the mathematics curriculum (Ainley and Pratt, 2010; Gal, 2004; Ministry of Education, 2007; NCTM, 2000; Schield, 2010; Watson, 2006). Schield (2010) argues that one of the most important goals for teaching statistics in schools is to prepare students to deal with the statistical information that increasingly impacts on their lives. More specifically, critical stance (Gal, 2004) the ability to take and evaluative stance with respect to statistical flaws and biases contained in media, marketing and financial reports is of vital importance in the quest for statistical literacy.

Additionally, schools are being asked to prepare students to be flexible thinkers, lifelong learners, and to manage complexities of an uncertain world (OECD, 2005; Ministry of Education, 2007). Critical thinking and critical literacies are embedded across the statements for key competencies, values and descriptions of learning areas in the new curriculum (Ministry of Education, 2007). Watson (2000) states that the

cross-curricular need for statistical literacy skills is recognised in many curriculum documents around the world. Students should be able to evaluate critically claims made in media reports, ask *worry questions* such as; is the graph drawn appropriately, is any information missing, does it distort trends in the data? Despite official inclusion of critical thinking and graphing in curricula, in some countries the teaching of critical sense has been slow to develop due to a number of factors.

The introduction of critical statistical literacy means that teachers themselves have to acquire new knowledge as well as teaching skills to meet the demands of the curriculum. There is evidence that suggests that the emphasis on critical thinking and contextual understanding, however, can present challenges for teaching and assessment (Garfield & Ben-Zvi, 2008; Monteiro and Ainley, 2004; Shaughnessy, 2007; Watson, 2006). For example, Monteiro and Ainley (2004, p. 362) state:

“The teaching and learning of graphing in ways which encourage the development of Critical Sense is a challenge for teachers who need to guide the pedagogical setting to situations in which statistically relevant aspects are discussed”

Research in New Zealand and overseas (Hill, Rowan & Ball, 2005; Hunter, 2010; Shaughnessy, 2007; Watson, 2006) has consistently acknowledged the importance of the teacher subject knowledge in student learning. Another reason for this discrepancy may be the debate amongst educationalists and curriculum developers on the nature of statistics and mathematics and best practice for instruction (Gal & Garfield, 1997; Moore, 1997; Scheaffer, 2006). Due to the nature of the discipline, statistics is fraught with contextual issues, whereas mathematics often strips off the context in order to abstract and generalise. To assist teachers, a framework has to be identified that will provide information about the cognitive skills, including critical thinking in socially-based curriculum approaches. I believe such a framework is likely to be dynamic in nature and can be viewed as a developmental sequence. This means that prior knowledge and experiences will influence current understanding and lead to the development of more complex interpretations of graphical representations.

The aim of this paper is to furnish teachers and researchers with a tool that can be used to scaffold and assess students' understanding of graphical representation. Specifically, it presents and describes a conceptual framework that could be used in the classroom. The paper is organised in three sections. First of all, I review and analyse research literature on tables and graphs from a range of educational levels and background. Second, based on the cumulative findings I present and discuss a framework specific to graphical representations. In the final section, some suggestions for teaching and learning and research are considered.

RESEARCH ON ANALYSIS AND INTERPRETATION OF TABLES AND GRAPHS

Curcio and Artzt (1997) stated that the ability to interpret and predict from data presented in graphical form is a higher-order skill that is important in our technological society. Curcio and Artzt's focus on graphical representations is consistent with the views of Gal (2004), Watson (2006) and the Ministry of Education (2007). A stated aim of the Mathematics in New Zealand Curriculum (Ministry of Education, 2007), for example, is to provide opportunities for students to interpret data in charts, tables, and graphs of various kinds. Gal (2004) suggests that since most students are more likely to be consumers of data rather than researchers, they need to learn to interpret results from statistical investigations and pose critical and reflective questions about reported data. Clearly reading, interpreting, and using data from tables and graphs are important cognitive skills for all.

A number of research studies from different theoretical perspectives seem to show that students are particularly weak in drawing inferences and predicting from tables and graphs (eg Bright and Friel, 1998; Curcio, 1987; Estepa, Bataneo, and Sanchez, 1999; Pereira-Mendoza and Mellor, 1991, Sharma, 1997). Kemp and Kissane report that although students can extract specific points of information, they do not critically interpret material in tabular or graphical form unless explicitly directed to do so. This section undertakes the task of reviewing the development of research with regards to data representation. A review of western studies will be provided first, and then attention will shift to research in non-western countries. This will be followed by a review of some emerging studies.

WESTERN STUDIES

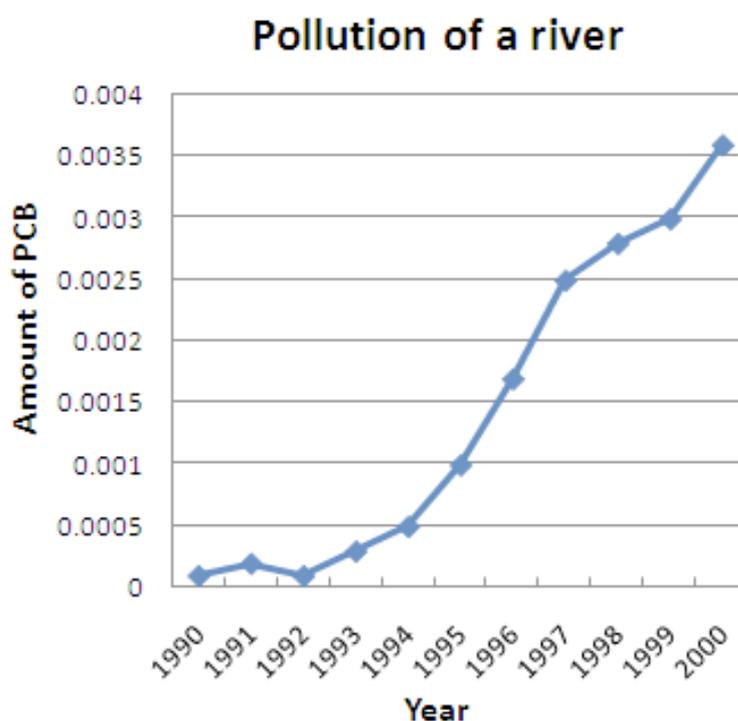
Curcio (1987) studied graph comprehension strategies of fourth and seventh grade students and identified three levels of graph reading which shed light on the complex nature of understanding graphs. These levels are:

- *Reading the data:* 'lifting' information to answer explicit questions for which the obvious answer is right there in the graph;

- *Reading between the data*: interpolating and finding relationships in the data presented in a graph;
- *Reading beyond the data*: extrapolating, predicting, or inferring from the representation to answer implicit questions (Curcio, p. 384).

As an illustration of the three categories, consider the graph of river pollution over time in Japan (Figure 1). In order to read the graph, students have to understand the scale and the measurement units, that this is a graph over time, 1990 to 2000 and the vertical axis (PCB) is an indicator of pollution level. Reading within the data is especially important for graphs over time in order to discuss general trends. For example, the timeline in Figure 1 shows a fairly steady increase in level of PCB with a couple of dips. Students can be asked to explain why there is a general increase or why did the dips occur. Reading beyond the data requires students to project into the future and answer questions about the graph. For instance, one may suggest that PCB level appears to be increasing at the end of the graph and that it will continue to increase in the future.

Figure 1: River Pollution over time in Japan



According to Curcio, the effect of prior knowledge about structural components of graphs (topic, mathematical content and graphical form) influences the reader's ability to comprehend mathematical relationships expressed in graphs. Curcio suggested that children should be actively involved in collecting the data and then describe the relationships and patterns that they observe in the sets of data that they have collected. These extended experiences will help students to read data competently.

Building on Curcio's work, Bright and Friel (1998) conducted a study of the ways that students in grades 6, 7, and 8 make sense of information in bar graphs. They explored ideas of reading the data, combining and comparing graphs and predicting from data on the following task.

A class of students has been collecting information about themselves. One question that they wanted to find out was how many children each person in class had in his/her family.

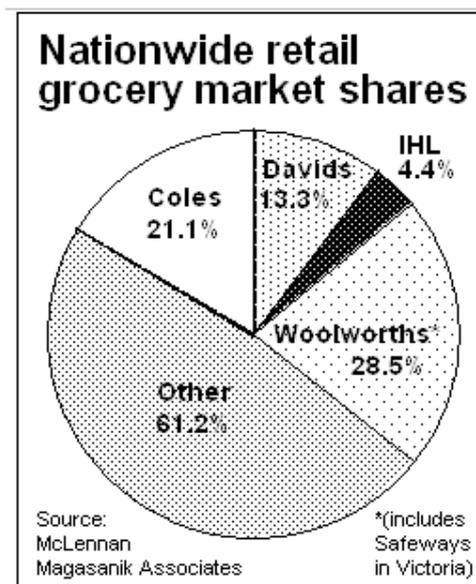
First, the subjects were shown a graph of unordered data and asked to find out how many children were in all the families in the class altogether. Next, the researchers gave them a new graph and asked their subjects to study this new graph for a few minutes and describe how they can find out how many students are in the class. The researchers reported that although these students had been exposed to many different bar graphs during both in-school and out-of-school experiences, they were not highly successful at answering questions that required higher order thinking skills. The students tended to want to move quickly to manipulation of information or seemed to be interpreting graphs in ways that were inconsistent with clear understanding of the underlying principles. The researchers claim that the process of data reduction and the structure of graphs are factors that influence graph knowledge. Bright and Friel

(1998) argue that tables may play an important role as an intervening representation that can smooth the transition between representing raw and reduced data.

Meanwhile, other writers have recognised that for students to effectively utilise graphs it is not sufficient for them to just be able to directly read information from a graph; they should be able to analyse and interpret from data and graphs in social settings. For instance, Pereira-Mendoza (1995, p. 2) notes:

“While drawing a graph and answering factual or low level interpretative questions are components of developing graphical skills, these are not the only components. In fact, in terms of the ability to use graphs in problem-solving situations or to analyse critically data in newspapers, on television or in other documents, these are the least important components”.

The above issues were addressed in some detail in the Pereira-Mendoza and Mellor study (1991). Here 121 grade 4 students and 127 grade 6 students were questioned on 12 different graphs, covering real/familiar topics such as height of children. Each graph consisted of three questions, a literal question, an interpretation question, and a prediction question. Although there were very few problems with the literal reading of graphs, there were major problems with the interpretation questions and the prediction questions. The analysis indicated two main sources of errors: data arrangement and the fact that the information was not shown on the graph. Another common difficulty students experienced was going beyond the graph; they could not give an answer because the information was not on the graph. The findings resonate with the findings of Shaughnessy and Zawojewski (1999) who report that students generally do much better on the literal reading of values on a graph than on tasks that require making inferences. Given the importance of statistical literacy for all students, judgement tasks should not be avoided just because they are difficult for students to answer and for teachers to assess. Most of the data displays encountered outside of school require some form of judgement, so helping students make sound judgements should be a teaching goal. Gal (1998) offered some helpful advice for posing and assessing judgement tasks. He recommended not giving specific hints about where to look in data displays on tasks requiring interpretation of tables and graphs. Questions should be worded to make clear that an opinion is called for rather than a specific number. He adds that questions should be set in familiar motivating contexts to ensure that students want to answer the question and use the full power of their reasoning. Gal recommended constructing rubrics for opinion tasks to facilitate the assessment process. He suggested that highest scores on the rubric may be awarded for responses that were reasonable in light of the given data, make correct use of terminology and make sensible assumptions about the sources of data. Watson and Moritz (1997) applied their three tiers of statistical literacy to analyse students' responses to some graphs taken from newspapers. An example of a misleading graph is given below.



What is the newspaper trying to tell its readers in this pie chart?

Watson and Moritz used the SOLO taxonomy of Biggs and Collis (1982) from developmental psychology to categorise statistical thinking into a three-tier hierarchy. In the first tier of Watson's model students

develop an understanding of the basic statistical and probability terms. In the second tier students are developing an understanding of statistical terms and concepts in context. At the most sophisticated level (tier 3) students are developing a questioning attitude and using critical thinking. Watson (2000) writes that the hierarchy of skills can be viewed as a progression of levels of statistical understanding and each tier in the hierarchy builds on the one before it. Watson and Moritz found gaps in students' understanding of the above graph within all three levels of statistical literacy. Only 10% of the students were aware of the complexity of or questioned some of the features of the graph. When students were asked to make some calculations based on the information in the pie graph, many of them ignored the graph altogether and reverted to personal experiences. In another study, Watson and Moritz (2001) identified three levels of student reasoning about pictographs. They also found cognitive conflict approach could improve student reasoning. Although the hierarchical tiers proposed by Watson and Moritz (1997) highlight the importance of statistical and mathematical knowledge in the interpretation of graphical displays, their categorisation does not take into account the graph reader's informal knowledge.

Estepa, Bataneo, and Sanchez (1999) studied how senior secondary students (17 to 18 year-olds) compared two small data sets with numeric values presented in tables. A questionnaire was completed by 213 and responses analysed from different points of view. One of the questions was based on ten before-after measurements of blood pressure involving a medical treatment, and the other was based on blood sugar levels of 10 boys and 10 girls. These authors classified the students' strategies into three categories; correct, partially correct and incorrect strategies. Some of the correct strategies were: comparing means, comparing totals, and comparing the two distributions. The partially correct strategies included finding out the difference for all pair wise cases and taking into account exceptional cases. Three incorrect strategies included comparing highest and lowest values, comparing ranges and basing conclusions on previously held theories or beliefs about the context of the data set.

On another task, bronchial disease and smoking, students were required to interpret a two-way tables. Students gave several different types of responses. Some students completely disregarded the table and opted instead to rely on previous theories about the dependence of illness on smoking. This sort of response reflects a somewhat extreme focus on context. The findings illustrate that although contextual knowledge can play a helpful role in data interpretation, it can also be problematic.

Based on analysis of literature on students' understanding of graphs, Friel, Curcio and Bright (2001) identified six behaviours (read, describe, interpret, analyse, predict and extrapolate data that they considered to be closely related with graph sense. Each of these elements seems to fit nicely with one of Curcio's three levels of graph reading discussed above.

- Recognising components of graphs (Reading the data)
- Speaking the language of graphs (Reading the data)
- Understanding relationships among tables, graphs and data (Reading within data)
- Making sense of graph but avoiding personalization and maintaining an objective stance while reading the graphs (Reading within data).
- Interpreting information in a graph and answering questions about it (Reading beyond data)
- Recognising appropriate graphs for a given data set and its context (Reading beyond the data)

Monterio and Ainley (2008) argue that familiarity with the above components is not sufficient to ensure understanding of specific graphs. They claim that that the context may be the key factor in understanding the comprehension of graphs. According to Monterio and Ainley, data displays used for analysis purposes are predominantly tools for detection of important or unusual features in the data. On the other hand graphs used for communication are defined as pictures intended to convey information about numbers and relationships among numbers. The authors add that the use of the kinds of "school graphs" which were used within Friel et al's study (for example displaying information about "the number of letters in students' names" or "how many raisins in various boxes) have limited purpose, in terms of analysing or communicating information which relates to interesting problems. Moreover, Monterio and Ainley state that in the specific examples used by Friel et al, the term *looking beyond the data* does not imply a need to look critically at the data and ask worry questions (see Gal, 2004). Indeed we might look beyond the data (extrapolating, predicting, or inferring from the representation) without being prompted to question the main idea presented in the data display.

Shaughnessy (2007) argues for another level of graph comprehension beyond Curcio's three levels that is reading **behind** the data or graph. He claims that since statistics is numbers in context, graph reading involves making connections between the context and the graph. This involves reading behind the graph. For example, in the river pollution graph (Figure 1) , one might look at historical, economical or

Royal Movie Theaters

								X					X						
								X					X						
								X	X				X	X					
								X	X		X	X	X	X					
								X	X	X	X	X	X	X					
5	6	7	8	9						1	1	1	1	1	1	1	1	1	4
										0	1	2	3						

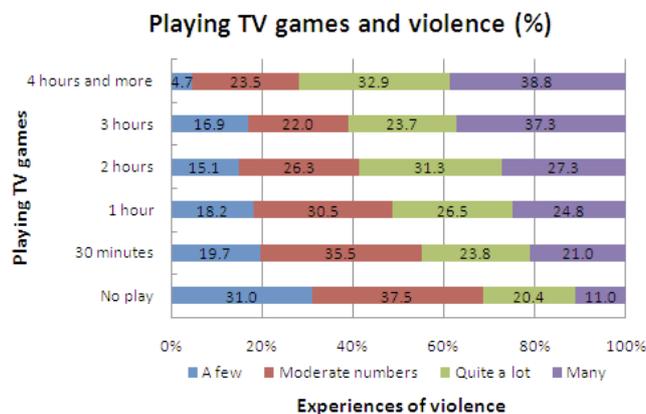
1. What can you conclude about the wait-times for the two theatres?
2. One student in the class argues that there is really no difference in wait-times for movies in both theatres, since the averages are the same. Do you agree or disagree? Why?
3. Which of these theatre chains would you choose to see a movie in? Why?

NON-WESTERN STUDIES

All the research discussed above has been done in western countries. Shaughnessy (2007) and Greer and Mukhopadhyay (2005). raised concerns about the lack of research in statistics education outside of western countries. Shaughnessy advocated large and small scale studies that examined group and cultural differences on students’ thinking in decision making. Indeed it would be interesting to determine how culture influences students conceptions of graphs, whether biases and misconceptions discussed in literature are artifacts of western culture or whether they vary across cultures. Moreover, with the large numbers of ethnic groups in mainstream classrooms all over the world with blanket statistics programs, it is important to actually listen to student voices to understand what or what may not work for these students in terms of their statistical thinking. Aoyama (2007) and Sharma (1997) studies are exceptions. Aoyama (2007) explored graphical interpretation component of statistical literacy in 175 participants from different educational levels (junior high school to graduate students) in Japan. Participants responded to a questionnaire with four items and some of them were also interviewed. Each item had a range of questions with four choices (see figure 4). Students had to choose an option and explain their answer. Rasch model (Watson and Callingham, (2003) was used to analyse the coded data to identify a hierarchy in the students’ responses. Five different levels of interpretations of graphs were identified: Idiosyncratic (level 1), basic graph reading (level 2), rational/literal (level 3), critical (level 4) , and hypothesiing and modeling (level 5). While a majority of students performed at level 3, a small number were operating at levels 4 or 5. Aoyama writes that statistics education in Japan mainly focuses on reading graphs (levels 2 and 3). Aoyama attributed students’ performance at level 4 to interpreting graphs in other learning areas such as sciences or experiences from non-school contexts.

Figure 4: Example of a test item

The following graph is the result of research that investigates how many hours per day elementary students play TV games at home and how many experiences of violence (e.g. hitting or pushing a classmate, or pulling someone’s hair) they have.



1. What is the percentage of students experiencing “Quite a lot” of violence in the group playing a TV game for one hour?
2. As the hours of playing a TV game increase, is the proportion of people experiencing “a few” violence also increasing?
a: Increasing; b: Decreasing; c: Do not change; d: I don’t know
3. As the hours of playing a TV game increase, is the proportion of people experiencing “many” violence also increasing?
a: Increasing; b: Decreasing; c: Do not change; d: I don’t know
4. It is suggested that a long time spent playing TV games will cause violence. Do you agree with this opinion?

As part of a larger study (Sharma, 1997) used two tasks to explore high school students’ understanding of tables and graphs. The secondary school selected for the research was an average high school in Fiji. The class consisted of 29 students aged 14 to 16 years of which 19 were girls and 10 were boys. Fourteen students were selected from this class and this group was representative of the larger group in terms of abilities and gender. The students were interviewed by the researcher and interviews were tape recorded and transcribed for analysis.

The task comparing temperatures of Ba and Sigatoka (Item 1) was used to elicit the students’ ideas about interpreting tables. The first question about whether Ba is warmer than Sigatoka was used to explore if students could read tables, and the second *What else do these figures reveal about the temperatures in Sigatoka and Ba?* attempted to find if students could make predictions from tables. In contrast to the first question, this question requires engagement with the overall pattern of data and consolidation of mathematical and statistical understanding.

Item 1: Task comparing temperatures of Ba and Sigatoka

Temperatures (in degrees C) were taken from Sigatoka and Ba on six consecutive days.

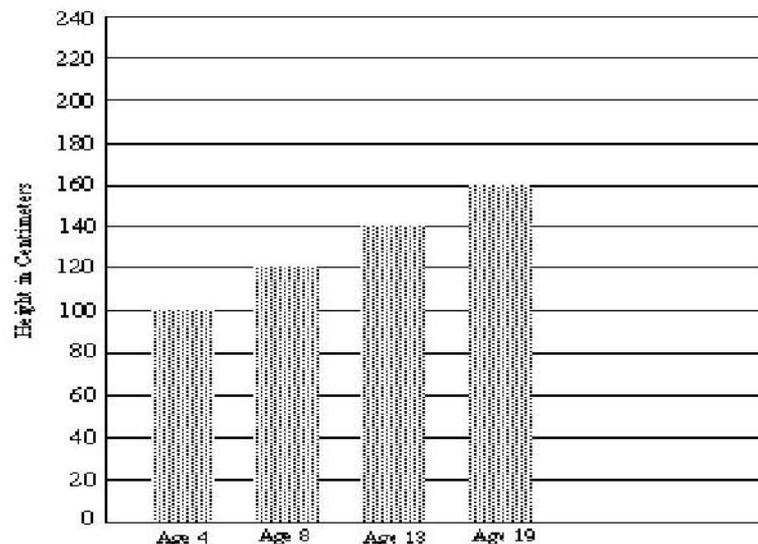
1. Look at the temperatures from both the towns and decide if Ba is warmer than Sigatoka. How do you know? Can you explain your answer?
2. What else do these figures reveal about the temperatures in Sigatoka and Ba? Can you summarise the information in another way? Can you explain your answer?

Day	1	2	3	4	5	6
Sigatoka	25	24	21	20	23	24
Ba	28	27	26	25	29	30

Item 2: Height of Sharma children

The following graph shows the height of four of the Sharma children, ages 4, 8, 13, and 19.

- (i) How tall is the 4 year old? How do you know? How much shorter is the 4 year old than the 19 year old? How did you work that out?
- (ii) A fifth child in the family is 10 years old. Can you tell how tall the 10-year-old is? Explain your answer.



I created a three category rubric to describe the results. The three categories in the rubric are: non-statistical, partial-statistical and statistical. The non-statistical responses were based on beliefs and experiences while the students using the partial-statistical responses applied rules and procedures inappropriately or referred to intuitive strategies.

Five students used non-statistical responses for both the questions. The non-statistical category consisted of students who mostly related the data to their everyday experiences in non-statistical ways. Of the five students who gave non-statistical replies on the first task, four based their reasoning on their everyday experiences. The students said Ba was warmer than Sigatoka and when asked to explain their answer talked about the everyday weather conditions of Ba and Sigatoka. Student 21 appeared to be inconsistent in his explanations.

Each day the temperature for Ba is greater than the temperature for Sigatoka. And normally Sigatoka is called a valley. They are producing fruits; it rains there. My one uncle is there. He mainly plants Chinese cabbages; because of the rain it grows so well there.

The initial answer (in the first sentence) is a reasonable response. However, in the next sentence the student goes off tangent. The student ignores the thrust of the question, which is about comparing temperatures of two towns and instead talks about weather conditions. This response reflects a somewhat extreme focus on context.

Although this study provides evidence that reliance upon previous experience can result in biased, non-statistical responses, in some cases this strategy may provide useful information for other purposes. For example, student 21's knowledge of geography may have been reasonable. The student has drawn on relevant common sense information. Gal (1998) suggests that such opinions constitute what students know about the world, they cannot be judged as inappropriate until a students' assumptions about the context of the data are fully explored. The responses raise further questions. Is there a weakness in the wording of this task in that it is completely open-ended and does not focus the students to draw on other relevant knowledge? For instance, the term *summarise* could have presented a linguistic problem as there are two perfectly reasonable interpretations.

Students' responses that were classified as partial-statistical on the task involving interpretation of tables (What else do these figures reveal about the temperatures of Sigatoka and Ba?) simply repeated the responses they had given for the first item. Some looked at the data and made some type of visual comparison. Unlike the task involving literal reading of the table (Item 1), none of the students' responses were classified in the non-statistical category for the bar graph. Although the responses of all 14 students were classified as statistical on the first question, only five students did not attempt to impose a pattern or give a specific numerical answer on the second. These five realised that their answer could not be an absolute number but would have to be expressed in some provisional way. Like the interpretation of tables (Item 1), one explanation for these differences could be a lack of emphasis in classrooms on interpreting graphs. Since the students lacked experiences in interpreting graphs, their responses were more likely to be in the partial-statistical categories.

Half the students were classified as using partial-statistical approaches for the second item. When rules were applied inappropriately, non-existent patterns imposed or no patterns seen, or exact answer to the question given, the responses were categorised as partial-statistical. Misinterpretations were caused by students forcing a pattern on the graph or not seeing the pattern. When asked to predict the height of the 10-year-old, three students tried to force a pattern on the bar graph. Student 14 justified this pattern in terms of a *going up* explanation. She said that it might be 130 cm because the *first one is 100cm and the 8 year old is 120cm. The 10-year-old might be 130cm. The trend continues, 100, 120 and 130.* The other two students cited the absence of a pattern as the reason for their inability to predict. This occurred even in cases where any attempt to search for a pattern made no conceptual sense. The students believed that a pattern must exist and consequently their inability to find the pattern resulted in their failure to offer any prediction. For example, when asked to predict a ten-year-old's height, student 20 said that he could not predict since there was no pattern on the *x-axis*. During the interview, the student continued to protect his flawed thinking rather than admit something was wrong.

As mentioned earlier, the results reported in this section were part of a larger study which focused on a number of areas of statistics. Since there had been virtually no research focused on graphing outside western countries, it was not clear when this study was conducted that the questions discussed in this section would be as rich and interesting as they were. For example, the tasks only highlighted technical aspects of data representation. The next section discusses some emerging studies which addressed some of these issues.

Emerging Studies

In preparing for a design experiment, Sharma et al. (2011) conducted whole class performance assessments with two groups of year 9 from the same school in which they planned to work. The assessment was undertaken by students in normal classroom settings rather than under test conditions. The students were told that the purpose of the assessment was to indicate how they use statistical information in everyday life situations. The students were required to read and think carefully about the various situations and if they were unsure of what to do they could ask for assistance. The instrument consisted of 8 tasks, each with a series of questions. Three tasks are used to discuss the finding here.

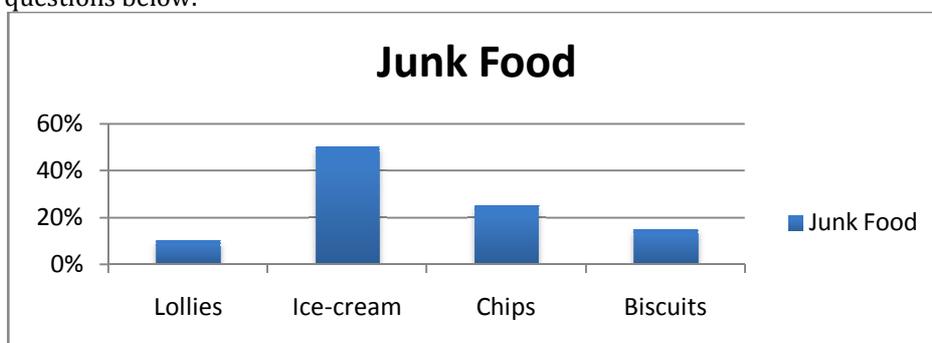
Question 1 displayed information about children's favourite junk foods in a bar graph. It required students to read information from the graph through to explaining their responses and asking worry questions.

In the comparing temperatures task (Question 2) students had to compare the temperatures in Auckland and Wellington and provide some explanations about how the temperatures change. They had to question how and why the data were collected and to think of the meaning within its context.

Question 3, called the race was set in the context of a championships. It addressed aspects of measures of centre and statistical variation. The open-ended question required students to make a choice using data provided in the form of a table and provide explanations for the choice.

Question 1 (Favourite Junk food)

The graph below shows information about children's favourite junk foods. Have a look at the graph and answer the questions below.



- What junk food did children say was their favourite junk food? Explain your thinking.
- What other information would you like before you can make decisions based on the graph? Explain your thinking.

Question 2 (Comparing temperatures)

Below are the temperatures (in degrees Celsius) on 12 consecutive days in Auckland and Wellington in September 2008.

Look at the temperatures from both the cities and answer the questions below.

Day	1	2	3	4	5	6
Wellington temperature °C	15	16	12	10	13	10
Auckland temperature °C	18	13	16	15	19	20

Day	7	8	9	10	11	12
Wellington temperature °C	13	15	9	13	8	12
Auckland temperature °C	13	11	15	15	16	15

- Is Auckland warmer than Wellington? How do you know?
- Have you got any questions about the information presented in the tables? Explain your thinking.

Question 3 (The 100 metre race)

The following table gives the times (in seconds) that each girl has recorded for seven of 100 metre races that they have run this year.

One girl is to be selected to compete in the upcoming championships.

RACE	1	2	3	4	5	6	7
Sarah	15.2	14.8	15.0	14.7	14.3	14.5	14.5
Rita	15.8	15.7	15.4	15.8	14.8	14.6	14.5
Maretta	15.6	15.5	14.8	15.1	14.5	14.7	14.5

Which girl would you select for the championships and why?

After the data were transcribed, constant comparison analysis (Strauss and Corbin, 1998) was used to interpret the responses to the survey questions. The parts of the tasks form a coherent task but were analysed independently so that a student could make an incorrect calculation but demonstrate understanding of the context. Details of the framework are provided in Figure 4.

Figure 4: Four Stages of Statistical Literacy Framework

At stages 0/1, the students were able to extract point information from the bar graph (Question 1: favourite junk food was ice-cream) and tables (Question 3: choose Rita because she has the highest). The students could find information by directly looking at the data display or comparing the data locally. However, there was no consideration of the context or data as a distribution. Random or no explanations or questions are likely to indicate reading/writing difficulties as the explanations could be lengthy and structurally complex. So students used random phrases such as *They both cold*. The response, *ice-cream because ice-cream is sweet* indicates lack of engagement with the problem context and use of non-statistical reasoning. This misinterpretation of the task may be related to the belief that the children like sweet junk food rather than survey students about favourite junk food. When asking questions, students focus on inappropriate or idiosyncratic aspects, for example, the question: *Why do we have to select girls can't it be mixed?* may be related to classroom activities where teachers use mixed ability grouping rather than focus on selecting a student for the championship (Item 3b). Questions asked reproduce the words used in the task, for example, *How many children don't like ice-cream? What time of the year were these recorded*

The responses also indicates a link to literacy skills for some students and the possible issues of reading a scenario.

At stage 2, responses indicate that statistical and literacy skills are sufficient for the problem to be understood but explanations focus on single features of data display or measures of centre, such as *Yes, the mean temperature of Auckland is 15.5C while the mean temperature of Wellington is 12C* without considering the need to integrate variability or context. Hence it is not just knowing curriculum based formulas such as add them up and divide by total number of values but integrating these with an understanding of the increasingly sophisticated settings within which questions arise.

At this stage, questions asked are likely to detect the critical features for representativeness or bias. For instance, *How many children were involved in the survey?* is judged as an appropriate question in this question because sample size can influence validity of findings

At stage 3, students start to appreciate many contexts although they cannot go further to explain/question data. In terms of questioning students present sample size, representativeness and random ideas such as *Was the survey random or systematic? Was the survey representative?*, however there is no evidence of integrating the statistical and contextual information.

At the top stage of the statistical literacy framework, students demonstrate critical thinking skills associated with sampling, measures of centre and data display. As mentioned previously, sophisticated statistical and higher order skills are associated with success at stage 4, specially in media contexts. For the junk food task students are likely to suggest random methods or random methods combined with representation such as *100 boys and 100 girls picked at random*.

Monterio and Ainley (2004) argue that the interpretation of graphs is a complex process in which different elements are involved. Specifically, they suggest three aspects, which are related to the idea of

critical sense: cognitive, affective, and contextual. The authors claim that cognitive, affective, and contextual aspects are interrelated during the interpretation of graphs. For example, cognitive aspects may encompass informal knowledge, such as that related to intuitions, which might be associated with beliefs and other kind of affective elements. Hence they see graphing as a skill in which people balance the influence between these three elements.

Monterio and Ainley (2004) explored critical sense in graphing of 118 school student teachers using a questionnaire before they took a data handling curriculum methods course in primary school mathematics. The questionnaire consisted of two items based on media context. They interviewed some volunteers on the same graphs given in the questionnaire. The interview questions were:

1. What is the total of number of deaths and serious injury per year? What is the lowest actual death and serious injury rate
2. Between 1994-1995, and 1997-1998, there was a decline in the number of deaths and serious injuries. Which period represents the greatest decline? Which years represent the highest and lowest number of deaths and serious injuries?
3. What is your prediction for death rate and serious injury in 2001? If the targets for 2000-2010 were met, what do you think the pattern might be for 2010-2020?

The episode below with Betty provides some examples of interpretations produced when they asked the “read beyond the data” question 1.

R – “What is your prediction for death rate and serious injury in 2001?”

B - In 2001... Well I don't know. It's... there doesn't seem to be a trend. It gone from 765 to 633... it's dropped down again, but then its gone up to 632 [year 2000], and here I'm presuming that this was the rate it wasn't the set target. I'm presuming but either way even if you look at the graph, it has gone up... (pp 364-365)

Monterio and Ainley (2004) the exchange above indicated that Betty is looking for any trends related to the increases and decreases between 1994 to 2000. She appeared to be concerned about the graphs structure which is related to cognitive aspects. They added that in the following transcript Betty continued referring to technical aspects involved in the graph, however she interpreted the graph based on her personal experience. Then, she started to conjecture about the social context in which the data might be related.

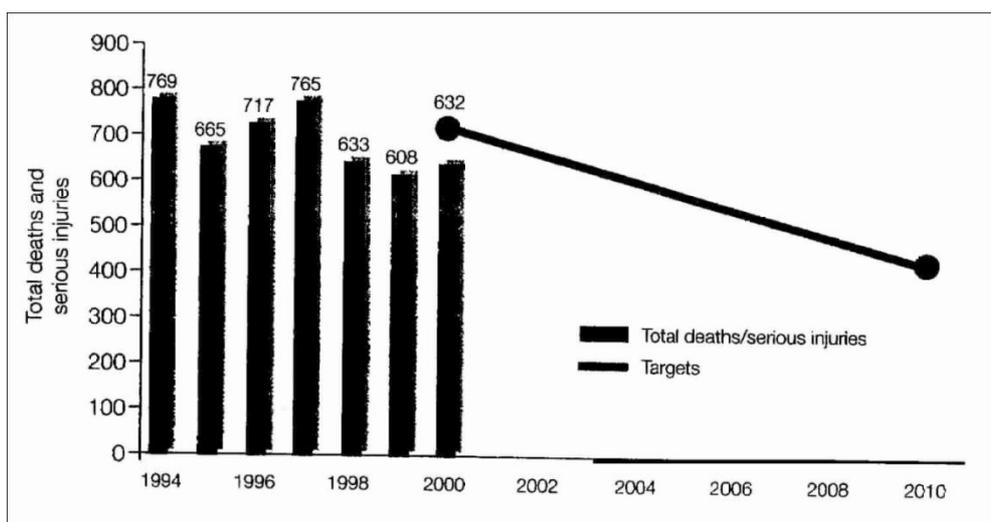
B...But throughout the whole period there hasn't been a set trend it dropped down[1994-1995], it's gone up [1995-1996], it's gone up [1996-1997], dropped down[1997-1998], dropped down [1998-1999], it's gone up [1999-2000]. If you based it on that... My husband is a currency trader; so all day is very boring he looks graphs all day. And he follows trends. That's how he buys and sells currency depending on trends. So He would look at this graph, he would say “are well the trend on it is to fluctuate” and he would draw lines in, and than he would say: “ok, dropped down, went up twice, dropped down twice, it's gone up”. Now he would plot a line to see, and he would go back over many years, he would look for the trend to see it follow a certain pattern. It is actually quite interesting. So he would want to look at more than this, he would probably say: “well maybe it would rise a little bit”. But, Again for me I don't have any information. I don't know what they're doing. Are they... you know... advertising more, trying to educate people, making people to wear seat belts, and things. It's hard to predict from these figures what's going to happen.

When encouraged to specify a prediction, she recognized that it was difficult but she made a prediction.

B - If I have to then, I would say it would rise slightly. Well if was 632 in 2000, maybe in 2001, if it rose there, maybe 640, something like that. But it's a guess. And I don't have enough information to be able to make a trending [moving with her hand like a curves of a graph, going up and down]... a trending estimate of it.

The researchers argue that Betty's reasonable answer was based on the articulation of cognitive, affective and contextual aspects about data presented on the graph.

Figure 5: graph reprinted from *Quality of life in Warwickshire*, 2001, pp. 93-94. (cited in Monterio and Ainley, 2004, p. 363)



SYNTHESIZING FINDINGS

This section is organized in two parts. First, the previous studies related to tables and graphs are briefly summarized. Second, I draw comparisons and critique across studies.

Summary of previous sections

Table 2 provides a summary of different researchers who investigated interpretation of tables and graphs, the population they studied and a summary of results from their work.

Table 2. Synthesizing Findings

Researchers	Population	Results
Curcio (1987)	Grades 4 and 7	Three levels of thinking were identified
Bright and Friel (1998)	Grades 6, 7, 8	Students experienced difficulty interpreting graphs that required higher order thinking skills
Pereira-Mendoza (1991)	Grades 4 and 7	Major problems with interpretation and prediction questions
Watson and Moritz (1997)	Grades 3, 6, 9	A three-tier hierarchy was developed
Friel, Curcio and Bright (2000)	Literature review	Six levels of thinking were identified
Estepa et al (1999)	Senior secondary students	Responses organized around correct and incorrect strategies
Shaughnessy et al. (2004)	Grades 6-12	Reading behind data
Aoyama (2007)	Junior high school to graduate students	Four levels of thinking identified
Sharma (1997)	Grades 10, 11	A three category rubric was identified
Monterio and Ainley (2004)	Pre-service teachers	Balancing cognitive, affective and contextual domains
Sharma et al. (2010)	Year 9	A Four stage framework was identified

Looking across studies

Curcio (1987) made an important contribution for understanding the processes involved in the interpretation of graphs. From student responses, Curcio identified three main types of interpretations; reading the data, reading within data and reading behind the data. However, Curcio only investigated the kinds of graphs used in school contexts and hence restricted the range of situations to which the interpretation of data display is connected. Curcio did not explore how students evaluated and criticized information displayed in tables and graphs. Moreover, the questions were forced-choice questions with no opportunity to query students as to the reason for their choices; thus, a missed opportunity to understand student thinking. Based on analysis of literature on students' understanding of graphs, Friel, Curcio and Bright (2001) identified six behaviours (read, describe, interpret, analyse, predict and

extrapolate data) that they considered to be closely related with graph sense. Monterio and Ainley (2004) claim that the familiarity with these processes is not sufficient to ensure understanding of a particular graph, the components of a graph might not be understood because context may be a key factor in understanding the elements of a graph.

Gal refers to two main type of contexts (situations in which data is interpreted), enquiry and reader. In enquiry contexts people act as data producers (eg statisticians and researchers) and usually interpret their own data whereas reader contexts emerge in everyday life situations such as watching television or reading newspapers and require people to evaluate someone else's others. He questioned the assumption that students who learn to process data (school settings) can transfer these skills to interpreting and critically evaluating statistical information (everyday situations). For Gal, statistical knowledge is contingent upon the context or setting. There is a need to develop a dynamic model of statistical literacy which focuses not only on cognitive skills but also on the dispositional aspects such as beliefs, values and attitudes.

Monterio and Ainley (2004) make a compelling argument that there is a significant difference between different versions of the tasks in terms of the type of reasoning involved. They argue that in the Curcio study the focus is on a composition because specific knowledge of graphing is not the only factor that supports the interpretation of media graphs.

Many of these studies did not use similar populations of students (e.g., university students and/or same countries). Student responses across ages and cultures contain significant sources of variability such as normal mathematical maturity as well as type of curriculum materials. Second, the sample sizes used in the studies reported here were all relatively small and none of the studies incorporated a randomized control study of different versions of the task. Thus, any broad conclusions from these studies should be tempered in light of this knowledge. Furthermore, as has already been mentioned, many of these studies did not ask students how they interpreted the problems or to explain their reasoning. In line with more recent trends in research we would need to interview students using different versions of tasks (literal reading versus contextual engagement) to have some sense of whether students indeed interpreted the questions differently and, if so, how.

More recent research suggests that problems related to interpretation of media graphs are indeed much more complicated than previously thought. For example, Monterio and Ainley (2004) claim that students, when asked to make predictions from media graphs, need to mobilise and balance several aspects involved such as mathematical knowledge, personal opinions and experiences, and affective component. Monterio and Ainley (2008) use the term mobilization to emphasise that when someone is engaged in interpreting a graph, he or she is not transferring knowledge and experiences from previous situations but triggering those elements for use in the current situation. I also believe that to be able to interpret data displays students need to demonstrate conceptual understanding by making connections between concepts (e.g. summary statistics, sample size, and variability), which Monterio and Ainley (2004) claim integrate formal knowledge with informal knowledge developed outside the class.

Outside the school context, the ability to look critically at the graph and the data presented, which we call *critical sense* is something more than reading graphs. Interpretation of graphs is a complex activity in which students establish relationships within data and infer information based on prior experiences. Cognitive, affective, and contextual aspects are interrelated during the interpretation of tasks. The framework below takes into account these three components. It is quite difficult to fit the students' responses into hierarchical classification which for example will make critical thinking the highest tier of complex statistical literacy. The boundaries between the stages are not hard edges but rather provide a set of stages that give a convenient way of describing changes as students progress to higher levels of thinking. The aim is to furnish teachers and researchers with a tool that can be used to scaffold and assess students' thinking.

A Five-stage framework to diagnose students' interpretation of tables and graphs

Stage 0: Informal/Idiosyncratic

Students at this stage are exhibiting characteristics of pre-structural thinking

- There is only an informal engagement with context, often reflecting intuitive non-statistical ideas and beliefs.
- Due to reading or writing difficulty, students are unable to explain their thinking often guess answers. With respect to statistical terminology, students provide random or inappropriate explanations. When making inferences, students focus on imaginative story telling or inappropriate aspects. Students use subjective reasoning to describe measures of centre or spread of data.
- Unable to recognise data values-confusion between scale and frequency values

- Questions asked are not based on the data or focused on irrelevant contextual issues.

Stage 1 Consistent non-critical (Reading data)

Students at this stage are exhibiting characteristics of pre-structural or at most uni-structural thinking

- Students are successful at some basic table and graph reading, as these require understanding of single elements and basic one-step straightforward reading.
- Case value attention to data
- Using memorized algorithms

Stage 2 Consistent non-critical (Reading within data)

Students at this stage are exhibiting characteristics of uni-structural and multi-structural thinking.

- Students focus on a single relevant aspect or attempt to attend to one or more relevant aspect of the data but have difficulty in integrating the aspects.
- Appropriate but non-critical engagement with context.
- Accurate use of statistical skills associated with simple statistics and graph characteristics.
- Single or partially correct comparisons made within a data table or graph.
- General or single statements made about the data collection methods and validity of findings with no reference to context.
- Questions asked are valid but based on one aspect of data.

Stage 3 Early critical (Reading beyond the data)

Students at this stage are beginning to exhibit characteristics of relational thinking. Students at this stage can attend to more than one relevant aspect of the data and are beginning to integrate the aspects.

- There is critical engagement in familiar contexts. There is selective engagement with unfamiliar contexts with some justification.
- There is appropriate use of terminology, qualitative interpretation of chance and appreciation of variation. Students demonstrate awareness of relevant features of displays, measures of centre and spread, however these are primarily based on the data or the context but not both.
- Questions asked of the data are based on more than one aspect of data task but not always connected.
- Students are likely to relate several elements together about data collection methods and graphing, they can manage two variables at the same time.
- Attention to entire distribution of data, including information about shape, centre and spread

Stage 4 Advanced Critical (Reading behind the data)

Students at this stage are integrating statistical and contextual knowledge that exhibits extended abstract thinking.

- There is a critical, questioning engagement with context.
- There is an understanding of the purpose of the data, data displays, measures of centre and inferences made. There is a critical evaluation of data collection methods, choice of measures and validity of findings that shows appreciation of variation and the need for uncertainty in making predictions.
- Sophisticated statistical and mathematical skills are associated with success at this stage, especially in media contexts.
- There is the ability to interpret subtle aspects of language.
- Connection between the graphical representation of quantitative data and the corresponding centre and variability for that data set
- Questions asked are based on relevant features of the data and the context using multiple perspectives.
- **Stage 5: Articulation of cognitive, affective and contextual aspects about data** (Reading behind the data)

DISCUSSION AND IMPLICATIONS

The early studies focused on telling us what student's natural tendencies regarding data displays were. The tasks were not designed to give us insight into how students think statistically, nor how they interpreted the task. Subsequent studies provided further insights into how students might interpret this task (or variations of it) as well as the reasoning behind their selections. On the one hand, worthwhile among the findings are that students reasoning changes over time (natural development from reading to

focus on procedures, to focus on contexts) and that students may have different intuitive interpretations of graphs in different settings. Can statistics educators use these studies to evaluate different theories and methods of education? In particular, the field has an initial framework for the kinds of reasoning students apply to this problem, including reasoning types at different grade levels (elementary school through graduate statistics students). This initial frameworks provide an initial starting point for future investigations that could refine, revise and extend the current frameworks. Furthermore, the frameworks and results of the various studies synthesized here can be of value to teaching. For example, if we know that young students tend to start off reasoning about these kinds of problems additively then we can sequence instruction to build toward proportional reasoning skills. Likewise, if we know that students may hold differing interpretations in different settings then we can address the underlying notions of data displays more explicitly in instruction.

All the studies discussed in this paper involved relatively small samples of students where researchers had convenient access to the populations. Thus, any results from these studies are tentative. Many of the conjectures and hypotheses outlined in the research are currently without a strong basis. Fortunately this provides an opportunity for future work to continue in this area to fill in the gaps left in these studies. Future research should begin with these studies to provide a roadmap. Implications for research and teaching are outlined below.

This paper into identifying and describing students' reasoning in regards to graphical representations has opened up possibilities to do further research at a macro-level on students' thinking and to develop more explicit descriptors for each stage of the framework discussed in this study. Much more understanding about students' thinking can be obtained when tasks allow for demonstration of several levels of student thinking. Such research would validate the categories of response types described in the current framework and raise more awareness of the levels of thinking that need to be considered when planning instruction and developing students' graph sense.

This paper describes five stage hierarchy characterised by increasingly complex processes in which both statistical processes and contextual understanding are involved. Contextual knowledge is an important component of this framework. The need for rich contextual experiences in statistical literacy raises the issue of a integrated approach to learning. Although this philosophical stance is not new in our curriculum, it seems that students will benefit from an integrated approach in teaching in which a topic is used as a focus not only for mathematics but also for other subjects such as science and language. By providing opportunities for the students to discuss, describe and investigate a topic like Eating junk food (Sharma, 1997 Item 1) a teacher could give students further exposure to the context which in turn would be advantageous when they undertake tasks like data evaluation and interpretation. Research is needed to investigate integrated curriculum approaches on statistical literacy learning especially processes like interpretation and critical literacy.

The use of open-ended tasks used in some of the studies (eg Monterio and Ainley, 2004; Sharma, 1997) gave students the opportunity to display their understandings in both school and everyday contexts. However, some gaps in the content covered with respect to topics in the curriculum and literature emerged when the overall findings were considered. For example, there were no items that explicitly investigated more complex graphs such as histograms and scatterplots and cause and effect arguments. Data are presented in a variety of contexts in tabular form. It would be interesting to explore how easily students can convert data to other forms such as graphs and vice versa. There were also issues with the wording and format of tasks used in some of the studies. Students certainly had difficulty interpreting the comparing temperatures task in Sharma (1997) study. Questions need to be structured to allow for varying degrees of difficulty. Some may build up interest through an extended series of questions whereas others are shorter but striking in their context. Such issues could be addressed in future research.

The participants in most studies (eg Sharma, 1997) were a fairly small non-random sample from one school. Thus, the findings, in particular the number of students who thought about tables and graphs in a particular way may or may not generalize to the population of secondary school students as a whole in Fiji. There is a need for more research with larger, more random samples with different backgrounds to determine how common these ways of thinking are in the general population. Both qualitative and quantitative methods can be used in the same study. Data can be gathered and quantified from results of surveys on tasks in large numbers but interviews can also be conducted with smaller number of students. Hypothesis generated about why students are answering survey questions in particular ways could be validated in detailed interviews.

Cognitive, affective, and contextual aspects are interrelated during the interpretation of tasks. As mentioned earlier, Sharma study focused on students' thinking about graphs before they had studied it as

a part of their senior secondary course work. There is a need for research that examines students' thinking about data displays after they have completed their senior secondary coursework. In particular, there is a need to examine the effects of different types of instruction on students' thinking in these areas. This will help clarify cultural issues as well as ambiguous effects of teaching.

In some studies (eg Sharma et al, 2010), while some pupils answered some questions with reference to the numerical values, others used their cultural knowledge on other items. It would be interesting to see how easily (or whether) students who argued on the basis of cultural knowledge on these questions could be persuaded to argue purely on the basis of the numerical data. Pratt (2005, p. 185) argues that if "children have seen the lack of explanatory power of their own ideas they would reconsider recently learned knowledge." Future research could incorporate this into the interview procedure to explore this issue in more depth across different contexts.

Many students were part-way to providing a complete explanation, but needed more detail or precision. Teachers need to assist students to express what they already know about statistics with more precise statistical language. In the course of discussions, comparison of several interpretations of the task may be made. This may lead to judgments about what might constitute a good explanation and draw attention to missing details and help students develop written and oral communication skills consistent with Gal (2004).

It may be quite easy to teach students how to extract point information from data in table and graphs (stage 1) but it may be more difficult to help them develop strategies to question how and why the data were collected, to make comparisons within and between categories and to think about the meaning of data in context. This is what the framework can help teachers do in conjunction with sound pedagogical teaching and learning of statistical concepts. It can provide a means for teachers to scaffold their students' thinking through the development of examples such as those illustrated in this paper.

Teachers need to reflect on the curriculum they serve for teaching. Is the curriculum structured around the study of statistical procedures or the process of statistical investigation? Thinking carefully about the levels of thinking can help avoid pitfalls that may occur when implementing different types of tasks.

Although teachers are in frontline in terms of curriculum implementing curriculum developers and writers of curriculum standards also have important roles to play in offering high quality experiences to students studying statistics. Curriculum materials and standards documents should not emphasise tasks with low levels of cognitive demand or deemphasize nonmathematical aspects of doing statistics. Curriculum materials and standards documents need to take care to maintain fidelity to the discipline of statistics and help students become critical consumers and producers of statistical information.

Appropriate graphical displays and critical questions can be extracted from media articles. These questions can be introduced in the primary curriculum as there is a need for children to begin to question statistical reports at an early age. Whitin and Whitin ((2003) claim that even kindergarten children can be encouraged to question statistical information. To discourage students from beginning too sceptical about statistics, it is important to provide examples where statistics are used correctly, not only to show the bad examples with which we are familiar with. Students should appreciate the contributions that statistical literacy can make in their lives.

We lack comprehensive overview of statistics in different kinds of societies and cultures. An insight into the educational consequences that may be drawn from such overview. In short we lack a well founded socio-cultural perspective of statistics education. While some studies have begun to appear but still we are rather sparse and far from systematic across significant societal practices. To provide a moving picture

It is hoped that the findings discussed in this paper will generate more interest in research with respect to data representation and ideas that students possess and factors that may impact on student learning. There is also a need to focus on documenting the challenges and difficulties that researchers face in the process of conducting international studies and how cultural factors can influence researcher activities and research results (Cao, Forgasz and Bishop; 2007). For example, Cao et al. found that the procedures for conducting surveys in China and Australia quite different due to cultural differences. Indeed we need to look for new ideas and develop more collaborative and cross cultural research between practitioners and researchers in the future if we are to improve outcomes for all our students. Research methods developed in one cultural setting may not be appropriate in another cultural context. Teachers, curriculum developers and researchers need to work together to find better ways to help all students develop critical sense in graphing.

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