Trapezoidal Hesitant Fuzzy Multi-Attribute Decision Making Based on TOPSIS

Pathinathan.T1, Johnson Savarimuthu.S2

1P.G and Research Department of Mathematics, Loyola College, Chennai-34
Email id: nathanpathi@hotmail.com
2Department of Mathematics, St. Joseph’s College of Arts and Science, Cuddalore-1
Email id: johnson22970@gmail.com

ABSTRACT
In this paper, the problems faced by the farmers with regard to oil seeds cultivation in Villupuram district are being discussed using fuzzy decision making tools. Through this paper, TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method is being utilized for the newly introduced Trapezoidal Hesitant Fuzzy Set (TRHFS) to solve the uncertainty in the decision making process.

Keywords: Multi-Attribute Decision Making, Hesitant Fuzzy Set, Trapezoidal Hesitant Fuzzy Set, Technique for Order Preference by Similarity to Ideal Solution, TOPSIS.

INTRODUCTION
Vagueness and uncertainty in the real world-Multi Criteria Decision Making problems make the situation very complex. This complexity and impreciseness pave way for inaccuracy into the criteria’s weights and evaluation values of alternatives and in turn make it for researchers very hard to quantify it. To reduce the vagueness, A.L Zadeh [18] introduced Fuzzy Sets in the year 1965. Fuzzy Sets has been emerged as one of the foremost theory which enumerates the qualitative data in the large scale. However, in Fuzzy Sets the membership degree of the element is represented by a single value between zero and one, and a major setback of Fuzzy Sets is that often single values do not convey information correctly. In such cases, the concept of Fuzzy Number has been introduced to quantify the qualitative arguments.

In some cases, the membership degree of an element is not a single value but a set of values. Such situations are managed by Hesitant Fuzzy Sets (HFSS). Hesitant Fuzzy Sets are first introduced by Torra [11] and it permits the membership degree of an element to be a set of several values between 0 and 1. Wang et al provided an outstanding approach with HFSSs to solve MCDM problems. After the pioneering work of Torra, the HFS has received much attention and has been used in decision making and clustering analysis. Xia and Xu [16, 17 and 21] studied the aggregation operators of Hesitant Fuzzy Sets and applied them to decision making.

In this paper, an extension has been made and developed by fitting the Trapezoidal Hesitant Fuzzy Set (TRHFS) approach with decision making package TOPSIS. The problems faced by the farmers in Villupuram District are analyzed through our newly designed Trapezoidal Hesitant Fuzzy Multi-Attribute Decision Making (TRHF - MADM) method.

This paper has been organized in the following manner. The concepts of HFS, TRHFS and some of its basic properties have been introduced in section 2. Section 3 presents the working algorithm for TOPSIS. In Section 4 an application which uses hesitant information is presented. Section 5 deals with the adaptation and description of the problem and finally the paper is concluded in Section 6 with the interpretation of the final results.

BASIC DEFINITIONS
2.1 Fuzzy Set
Let $E$ be the universal set, let $x$ be an element of $E$, then the fuzzy subset $A$ of $E$ is a set of ordered pairs

$$A = \{ (x | \mu_A(x)) \} \text{, for all } x \in E$$

(2.1)

where $\mu_A(x)$ is the grade (or) degree of membership of $x$ in $A$.

$\mu_A(x)$ takes the value from the membership set $M = [0,1]$ and $\mu_A(x)$ is the membership function or characteristic function.

### 2.2 Hesitant Fuzzy Set

Let $X$ be a fixed set, a Hesitant Fuzzy set (HFS) on $X$ is in terms of a function that when applied to $X$ returns a subset of $[0,1]$.

Mathematical representation of Hesitant fuzzy set [18]:

$$A = \{ < x, h_A(x) / x \in X \}$$

(2.2)

where $h_A(x)$ is a set of some values in $[0,1]$, denoting the possible membership degrees of the element $x \in X$ to the set $A$.

### 2.3 Hesitant Fuzzy Element (HFE)

Every $h = h_A(x)$ is defined as hesitant fuzzy element (HFE).

**Example** Let $X = \{ x_1, x_2, x_3, x_4 \}$ be a fixed set,

- $h_{A}(x_1) = \{0.2,0.4,0.5,0.4\}$,
- $h_{A}(x_2) = \{0.3,0.4,0.5\}$ and
- $h_{A}(x_3) = \{0.3,0.2,0.5,0.6\}$
- $h_{A}(x_4) = \{0.2,0.4,0.6\}$

be the HFEs of $x_i$ ($i=1,2,3,4$) to the set $A$ respectively. Then $A$ can be considered as a HFS:

$$A = \{ < x_1, \{0.2,0.4,0.5,0.4\} \}, < x_2, \{0.3,0.4,0.5\} \}, < x_3, \{0.3,0.2,0.5,0.6\} \}, < x_4, \{0.2,0.4,0.6\} \}$$

### 2.4 Special properties of HFS

- (i) Empty Hesitant Fuzzy Set: $h = \{0\}$
- (ii) Full Hesitant Fuzzy Set: $h = \{1\}$, denoted as $I^*$.
- (iii) Complete Ignorance: (All are possible) $h = [0,1] U^*$
- (iv) Nonsense Set: $h = \{0\}$

### 2.5 Score of an HFS

For a Hesitant Fuzzy Element (HFE) $h$,
Pathinathan and Johnson

\[ s(h) = \sum_{i=1}^{n} \frac{y_i}{l_i} \]  

(2.3)

is called the score of \( h \), where \( l_i \) the number of the elements in \( h \).

**Case(i):** For two HFEs \( h_1 \) and \( h_2 \) if \( s(h_1) > s(h_2) \), then \( h_1 \) superior to \( h_2 \), denoted by \( h_1 \succ h_2 \)

**Case(ii):** \( s(h_1) = s(h_2) \) then \( h_1 \) is indifferent to \( h_2 \), denoted \( h_1 \asymp h_2 \)

**Definition 2.6: Fuzzy Number**

A Fuzzy number \( A \) is a fuzzy set on the real line \( R \), which must satisfy the following conditions.

(i) \( \mu_A(x) \) is piecewise continuous

(ii) There exist at least one \( x_0 \in R \) with \( \mu_A(x_0) = 1 \)

(iii) \( A \) must be normal and convex

**Definition 2.7: Triangular Fuzzy Number**

Triangular Fuzzy Number is defined as \( A = \{a,b,c\} \), where all \( a, b, c \) are real numbers and its membership function is given below.

\[
\mu_A(x) = \begin{cases} 
0 & \text{for } x < a \\ 
\frac{x-a}{b-a} & \text{for } a \leq x \leq b \\ 
1 & \text{for } x = b \\ 
\frac{c-x}{c-b} & \text{for } b \leq x \leq c \\ 
0 & \text{for } x > c 
\end{cases}
\]

**2.8 Triangular Hesitant Fuzzy Set**

Let \( X \) be a fixed set, a Triangular hesitant fuzzy set (THFS) defined by

\[ A = \{< x, Th_A(x) > / x \in X \} \]  

(2.4)

where \( Th_A(x) \) is a set of some triangular values in \( (a_L, a_M, a_U) \).

**Definition 2.9: Trapezoidal Fuzzy Number**

A fuzzy set \( A = (a, b, c, d) \) is said to trapezoidal fuzzy number if its membership function is given by where \( a \leq b \leq c \leq d \)
2.10 Trapezoidal Hesitant Fuzzy Set (TrHFS)

Let $X$ be a fixed set, then a Trapezoidal Hesitant Fuzzy Set (TrHFS) $D$ on $X$ is described as:

$$D = \{ x, T_h(x) > x \in X \}$$

in which $T_h(x)$ is function of $T_h(x) = (a^L, a^{M_1}, a^{M_2}, a^U)$

2.11 Trapezoidal Hesitant Fuzzy Element (TrHFE)

The quadruple $(a^L, a^{M_1}, a^{M_2}, a^U)$ is called Trapezoidal fuzzy element

2.12 Hesitant Multiplicative Aggregation

To quantify the natural statements presented by the decision maker, we employed Saaty’s 1-9 scale with its respective meaning.

<table>
<thead>
<tr>
<th>1-9 scale</th>
<th>0.1-0.9 scale</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/9</td>
<td>0.1</td>
<td>Extremely not preferred</td>
</tr>
<tr>
<td>1/7</td>
<td>0.2</td>
<td>Very strongly not preferred</td>
</tr>
<tr>
<td>1/5</td>
<td>0.3</td>
<td>Strongly not preferred</td>
</tr>
<tr>
<td>1/3</td>
<td>0.4</td>
<td>Moderately not preferred</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>Equally preferred</td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
<td>Moderately not preferred</td>
</tr>
<tr>
<td>5</td>
<td>0.7</td>
<td>Strongly preferred</td>
</tr>
<tr>
<td>7</td>
<td>0.8</td>
<td>Very strongly preferred</td>
</tr>
<tr>
<td>9</td>
<td>0.9</td>
<td>Extremely preferred</td>
</tr>
<tr>
<td>Other values between 1 and 9 (2,4,6,8)</td>
<td>Other values between 0 and 1</td>
<td>Intermediate values used to present compromise</td>
</tr>
</tbody>
</table>

3. Trapezoidal Hesitant Fuzzy Algorithm Based on TOPSIS

This section puts forward a framework for determining the ranking orders for all the alternatives under hesitant fuzzy environment. The approach involves the following steps:

**Step 1:** We construct attribute decision making problem, let $H = \{h_1, h_2, ..., h_p\}$ be a set of alternatives, $X = \{x_1, x_2, ..., x_m\}$ a set of attributes then the Trapezoidal hesitant decision matrix

**Step 2:** Choose weight vector $W=(w_1, w_2, ..., w_m)^T$ where $w_i \in [0, 1]$ and $\sum_{i=1}^{m} w_i = 1$ for each attribute according to their importance over the problem.
Step 3: We can calculate the degree of similarity of the positive ideal and negative ideal by the following expression

\[ A^+ = \left< x_j^{\max} \{ trh_{ij} \} : j = 1, 2, \ldots, m \right> \]  
\[ A^- = \left< x_j^{\min} \{ trh_{ij} \} : j = 1, 2, \ldots, m \right> \]  

Step 4: Calculate the separation measures \( d_i^+ \) and \( d_i^- \) of each alternative \( A_i \) from the Trapezoidal hesitant fuzzy positive ideal \( A^+ \) and negative ideal \( A^- \) respectively by the following expression;

\[ d_i^+ = \left( \sum_{j=1}^{m} d(h_{ij}, h_j^-) w_j \right) \]  
\[ d_i^- = \left( \sum_{j=1}^{m} d(h_{ij}, h_j^+) w_j \right) \]  

Step 5: We find the relative closeness coefficient \( c(A_i) \) and corresponding to each alternative \( A_i \) to the hesitant fuzzy solution by using the formula;

\[ c(A_i) = \frac{d_i^-}{d_i^+ + d_i^-} \]  

Step 6: Rank the alternatives according to the relative closeness to the hesitant positive ideal and negative ideal solution.

4. Trapezoidal Hesitant Fuzzy Set Application

Pathinathan.T and Johnson Saravimuthu.S [22] studied the problem faced by the farmers who plant cash crops in the Villupuram district. In this paper, we extend our research work by analyzing oil seeds cultivation in the same locality. In Villupuram district, it has been observed that the farmers show a lot of interest in planting oil seeds such as groundnuts, palmli, sesame, sunflower. Oil seed cultivation depends on water resources from river and tanks. Rivers in Villupuram such as Pennaiyar, Kadilam, Komuki and Varahanathi serve as the major water source of the district and its Vidur, Sathanur, Komuki have stopped functioning they operate only during raining season.
The ground water level is very low due to the failure of the seasonal rains. Due to frequent power cut bore wells cannot be used. Farmers cannot afford to get loan from public banks and private money lending institutions to buy diesel for operating bore wells because it is not possible to pay back the debts. Faced with these problems, farmers from Thirukovilur, Kalpet and Arasur of Villupuram district have been selling their agricultural land to Real Estate agents for their business. Interviews were conducted to grasp the struggle of farmers in Villupuram district. The present study considers the hierarchical structure of four major cultivating oil crop alternatives with six major attributes which cause severe hindrance in production given away by the experts.

4.1 Experts (from Villupuram District)
We collected the overall information regarding agriculture problems faced in oil seed cultivation from the following Experts in Villupuram District.

**DM1.** Mr. S. Kannan (Farmer), Thirukovilur
**DM2.** Mr. S. Manikandan (Farmer), Kalpet
**DM3.** Mr. Muthuvel, (Farmer), Arasur

### 4.2 Alternatives

The following are the major oil seeds cultivated in Villupuram District. We took these activities as our alternatives.

- A₁. Groundnuts
- A₂. Palm lien
- A₃. Sesame
- A₄. Sunflower

### 4.3 Attributes

We recorded few major problems that hindered farmers’ lives through the interview and consolidated them into six major attributes. These attributes are evaluated into two forms, namely;

1. Benefit Type (Qualitative in nature)
2. Cost Type (Quantitative in nature)

- **X₁. Crop failure (Benefit Type)** – Nutritional need of the people is not met as crop yield has been getting reduced year by year. Crop failure is the result of drying crop and inability to salvage the standing crop due to water scarcity.

- **X₂. Crop debt (Cost Type)** – Money which is borrowed by a farmer such as debt and money borrowed from private money lenders to meet the expenses.

- **X₃. Lack of water (Benefit Type)** – Water scarcity in Sathanur dam and the truant behavior of monsoons.

- **X₄. Heavy rain and Cyclone (Nilam) (Benefit Type)** – Heavy rain causing soil erosion and soil fertility destroying much livelihood and farmlands in recent years is one of the natural calamities faced by every farmer in the district of Villupuram.

- **X₅. Lack of Electricity (Cost Type)** – Load shedding is one of the reasons for increased use of diesel engines but the cost of diesel is again a burden on the farmers.

- **X₆. Demand for Fertilizers and Pesticides (Cost Type)** – Scarcity of fertilizers and pesticides resulting hatred against the government.

### 4.4 Hierarchical Structure for TrHFS

The hierarchical structure of this decision making problem is shown from the below diagram;

5. Adaptation and Description of the problem:

Hesitant Fuzzy Decision Matrix is obtained by considering each and every Expert’s opinion with their possible membership values and they are recorded as follows:-
Similarly, the biggest burden for farmers who indulge in groundnuts cultivation.

For instance, experts are asked to give their opinions and their opinions are tabulated.

\[
A_i(x) = (\text{TrM}_1, \text{TrM}_2, \text{TrM}_3)
\]

where, 

\[A_i, i = 1, 2, 3, 4\text{, (four alternatives)}\]

\[X_i, i = 1, 2, 3, 4, 5, 6\text{ (six attributes)}\]

\[A_i(x), i = 1, 2, 3, 4, 5, 6\text{ denotes Alternative 1 (groundnuts) comparing with all six attributes. And all the three experts are asked to give their opinions and their opinions are tabulated.} \]

\[
A_i(x) = (0.6, 0.7, 0.8, 0.9)
\]

nine attributes. And all the six attributes.

Similarly, \(\text{TrM}_2\) provide 0.4 as the membership value and so on...

Suppose, if \(A_4(x) = (0.3, 0.5, -0.5)\), denotes on discussing Alternative 2 (palmlien) with the attribute 1 (crop failure), \(\text{TrM}_3\) fail to record his value due to unfamiliarity about the respective alternative over the attribute.

\[
\begin{array}{ccccccc}
\text{X}_1 & \text{X}_2 & \text{X}_3 & \text{X}_4 & \text{X}_5 & \text{X}_6 \\
\hline
A_1 & (0.8, 0.4, 0.6) & (0.8, 0.5, 0.6) & (0.6, 0.4, 0.1) & (0.6, 0.4, 0.1) & (0.4, 0.1, 0.1) & (0.9, 0.9, 0.9) \\
A_2 & (0.3, 0.5, 0.3) & (0.7, 0.8, 0.4) & (0.5, 0.5, 0.1) & (0.6, 0.4, 0.4) & (0.4, 0.1, 0.1) & (0.9, 0.9, 0.9) \\
A_3 & (0.4, 0.4, 0.4) & (0.8, 0.7, 0.5) & (0.5, 0.3, 0.1) & (0.6, 0.4, 0.1) & (0.4, 0.1, 0.1) & (0.9, 0.9, 0.9) \\
A_4 & (0.1, 0.1, 0.8) & (0.8, 0.7, 0.4) & (0.4, 0.3, 0.1) & (0.6, 0.4, 0.1) & (0.4, 0.7, 0.4) & (0.9, 0.9, 0.9) \\
\end{array}
\]

Table 5.4: Positive Ideal Solution (by Eqn: 3.1)

<table>
<thead>
<tr>
<th>Positive Ideal Solution</th>
<th>Negative Ideal Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_1^*) &lt; (x_1; (0.6, 0.7, 0.8, 0.9))</td>
<td>(A_1^-) &lt; (x_1; (0.1, 0.2, 0.3, 0.4))</td>
</tr>
<tr>
<td>(A_2^*) &lt; (x_2; (0.6, 0.7, 0.8, 0.9))</td>
<td>(A_2^-) &lt; (x_2; (0.2, 0.3, 0.4, 0.5))</td>
</tr>
<tr>
<td>(A_3^*) &lt; (x_3; (0.4, 0.5, 0.6, 0.7))</td>
<td>(A_3^-) &lt; (x_3; (0.1, 0.1, 0.2, 0.3))</td>
</tr>
<tr>
<td>(A_4^*) &lt; (x_4; (0.4, 0.5, 0.6, 0.7))</td>
<td>(A_4^-) &lt; (x_4; (0.1, 0.1, 0.2, 0.3))</td>
</tr>
<tr>
<td>(A_5^*) &lt; (x_5; (0.5, 0.6, 0.7, 0.8))</td>
<td>(A_5^-) &lt; (x_5; (0.1, 0.1, 0.2, 0.3))</td>
</tr>
<tr>
<td>(A_6^*) &lt; (x_6; (0.9, 0.9, 0.9, 0.9))</td>
<td>(A_6^-) &lt; (x_6; (0.9, 0.9, 0.9, 0.9))</td>
</tr>
</tbody>
</table>

Then, by utilizing the equation (3.3 and 3.4), we have the following distance values:
Hamming Distance Calculations,

\[ d_1^+ = 0.25 \sqrt{\frac{0.2 - 0.6^2 + 0.3 - 0.7^2 + 0.4 - 0.8^2 + 0.5 - 0.9^2}{3}} \]
\[ + 0.20 \sqrt{\frac{0.3 - 0.6^2 + 0.4 - 0.7^2 + 0.5 - 0.8^2 + 0.6 - 0.9^2}{3}} \]
\[ + 0.15 \sqrt{\frac{0.1 - 0.4^2 + 0.1 - 0.5^2 + 0.2 - 0.6^2 + 0.3 - 0.7^2}{3}} \]
\[ + 0.15 \sqrt{\frac{0.1 - 0.5^2 + 0.1 - 0.6^2 + 0.2 - 0.7^2 + 0.3 - 0.8^2}{3}} \]
\[ d_1^+ = 0.3978 \]

\[ d_1^- = 0.25 \sqrt{\frac{0.6 - 0.1^2 + 0.7 - 0.2^2 + 0.8 - 0.3^2 + 0.9 - 0.4^2}{3}} \]
\[ + 0.20 \sqrt{\frac{0.6 - 0.2^2 + 0.7 - 0.3^2 + 0.8 - 0.4^2 + 0.9 - 0.5^2}{3}} \]
\[ + 0.15 \sqrt{\frac{0.4 - 0.1^2 + 0.5 - 0.1^2 + 0.6 - 0.2^2 + 0.7 - 0.3^2}{3}} \]
\[ + 0.15 \sqrt{\frac{0.4 - 0.1^2 + 0.5 - 0.1^2 + 0.6 - 0.2^2 + 0.7 - 0.3^2}{3}} \]
\[ + 0.15 \sqrt{\frac{0.2 - 0.1^2 + 0.3 - 0.1^2 + 0.4 - 0.2^2 + 0.5 - 0.3^2}{3}} \]
\[ d_1^- = 0.3984 \]

Table 5.5: Hamming Distance for HFS (by Eqn: 3.3)

<table>
<thead>
<tr>
<th></th>
<th>P-Distance</th>
<th></th>
<th></th>
<th>N-Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_1^+ )</td>
<td>0.3978</td>
<td>( d_1^- )</td>
<td>0.3984</td>
<td></td>
</tr>
<tr>
<td>( d_2^+ )</td>
<td>0.4498</td>
<td>( d_2^- )</td>
<td>0.2606</td>
<td></td>
</tr>
<tr>
<td>( d_3^+ )</td>
<td>0.3978</td>
<td>( d_3^- )</td>
<td>0.2634</td>
<td></td>
</tr>
<tr>
<td>( d_4^+ )</td>
<td>0.0441</td>
<td>( d_4^- )</td>
<td>0.4157</td>
<td></td>
</tr>
</tbody>
</table>

By using equation 3.5, we recorded the closeness values among the alternatives and they are tabulated as follows:
REFERENCES

observed cultivation are the two crops, when compared preference ranking order relation as By aggregating

Table 5.6: Closeness for HFS (by Eqn: 3.5)

<table>
<thead>
<tr>
<th></th>
<th>(d_i^r)</th>
<th>(d_i^r + d_i^s)</th>
<th>(c(A_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c(A_1))</td>
<td>0.3984</td>
<td>0.7962</td>
<td>0.5003</td>
</tr>
<tr>
<td>(c(A_2))</td>
<td>0.2606</td>
<td>0.7104</td>
<td>0.3668</td>
</tr>
<tr>
<td>(c(A_3))</td>
<td>0.2634</td>
<td>0.6612</td>
<td>0.3983</td>
</tr>
<tr>
<td>(c(A_4))</td>
<td>0.4157</td>
<td>0.8569</td>
<td>0.4851</td>
</tr>
</tbody>
</table>

From the above table we rank the alternatives \(A_i\) \((i = 1,2,3,4)\) as:-

\[ A_1 \succ A_4 \succ A_3 \succ A_2 \]

CONCLUSION

By aggregating the opinion collected from the three Decision Makers from the table 5.6, we have the preference ranking order relation as \(A_1 \succ A_4 \succ A_3 \succ A_2\). (i.e.) Alternative \(A_1\) (groundnuts) is dominated by all the other alternatives. Sunflower \(A_1\) and Sesame \(A_3\) almost share the same ordering position when compared with the other alternatives. It shows that groundnuts cultivation and Sunflower cultivation are the two crops, which secure good yield, income and livelihood to farmers in terms of the observed attributes.

REFERENCES


