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Extended Generalized Gamma Function and Same Its Applications

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ABSTRACT

An extended form generalized gamma function is defined by slightly modifying the form of Kobayashi's generalized gamma function (1991). Its possible application in reliability theory, to study displacement phenomenon of the corrosion problem in a new machine or metal fatigue, is discussed. A few well known probability distributions are shown to be its particular cases. Hazard function, mean, variance etc are worked out for generalized gamma distribution. This extention open infinitely divisible distribution function and the density of its cases, since they are extended generalized gamma convolutions.

This paper derives a new family of Extended generalized gamma model function (EGGMF) based on the extended of Euler gamma function. Unusually, the extended transformation constrains the generalized function parameters algebraically but not linearly. Its consequences for EGGF are explored. From this formula new evaluations of six parameters with model form are derived, by applying transformations in the different form models. Their parameters are also constrained, and can be derived from as well.

The extended $\Lambda_r(k,p,m,n,\lambda)$ *give an important role in applications of convexity to such diverse fields as algebraic dynamics of the Gamma function stability of parametric constraint systems, and its applications to concrete problems such as finding equilibrium prices in mathematical economics, or hydrothermal scheduling. Its study is not only interesting but important, both, because most of the sex parameters values are special cases of* $\Lambda_r(k,p,m,n,\lambda,\alpha(x))$*, and because it is challenging to study a function whose formulation is so indeterminate. Recommended to study the special case of the model formula* $\alpha(x)=(x-a)/b$*, we summarize the recent main results about study of* $\Lambda_r(k,p,m,n,\lambda,a,b)$*, including definition, basic properties, monotonicities, comparison, generalizations of concepts of values, applications to theory of special functions. The development of computational techniques and the rapid growth in computing power have increased the importance of the special functions and their formulae for analytic representations. However, problems remain, particularly in heat conduction, astrophysics, and probability theory, whose solutions seem to defy even the most general classes of special functions.*

Key word*: Euler gamma function, recurrence formula, integral expression, density function, special functions, Stirling formula, extended gamma functions, completely monotonic functions, mixture, diffraction theory.*

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INTRODUCTION

The history of the gamma function is primarily due to Euler (1707-1783) who developed the analytical formulation of this function, since the famous work of J. Stirling (1730) who first used series for $\log(n!)$ to derive the asymptotic formula for $n!$ [2]. The gamma function was first introduced by the swiss mathematician Leonhard Euler (1707-1783) in his goal to generalize the factorial to non integer values.

Later, because of its great importance, it was studied by other eminent mathematicians like Adrien-Marie Legendre (1752-1833), Carl Friedrich Gauss (1777-1855), Christoph Gudermann (1798-1852), Joseph Liouville (1809-1882), Karl Weierstrass (1815-1897), Charles Hermite (1822-1901) as well as many others [1].

The problem of interpolating the values of the factorial function was first solved by Daniel Bernoulli and later studied by Leonhard Euler in the year 1792. The interpolating function is commonly known as the Gamma function [2].

Over the past 300 years there has been a substantial increase in the use of special functions in the formulation of solutions for scientific and engineering problems. Special functions are used as mathematical models for many and varied physical situations, and special functions also occur as reformulations of other mathematical problems [1].

In recent years, the theory of gamma and beta functions has been used in many different areas of mathematics and applied sciences and engineering. Many problems in the field of ordinary and, scattering theory in quantum mechanics [7], biology viscodynamics fluids, contact problems in the theory of elasticity, mixed boundary problems in mathematical physics, physical chemistry and engineering can be formulated as special functions [8].

As a matter of fact, it was Daniel Bernoulli who gave in the 1729, the first representation of an interpolating function of the factorials in form of an infinite product, later known as gamma function $\Gamma(x)$ [13].

The correspondence between Goldbach, Daniel Bernoulli and Euler which undoubtedly gave birth to the gamma function is well documented in Paul Heinrich Fuss's "Correspondance mathématique et physique de quelques célèbres géomètres du XVIIIeme siècle ", St. Pétersbourg, (1843) [25].

The gamma function $\Gamma(x)$ is the most important function in some areas, such as probability and statistics you will see the gamma function more often than other functions that *are* on a typical calculator, such as trig functions. A gamma function is the solution to a specific integral. Though the gamma functions, it is useful for physical applications [22].

It is occasionally related to the "error functions." Its simplest expression is at positive integer values, where it is the same as the factorial function. Which is factorial is the product of the integer in question with all positive integers smaller than that integer. Many of its other forms are recursive as well.

The gamma function extends the factorial function to real numbers. Since factorial is only defined on non-negative integers, there are many ways you could define factorial that would agree on the integers and disagree elsewhere. But everyone agrees that the gamma function is “the” way to extend factorial. Actually, the gamma function $\Gamma(x)$ does not extend factorial, but $\Gamma(x+1)$ does [25].

In a sense, $\Gamma(x+1)$ is the unique way to generalize factorial Harald Bohr and Johannes Mollerup year proved that it is the only log-convex function that agrees with factorial on the non-negative integers. Leonhard Euler is considered one of the top ten mathematicians in human history. He was an extremely prolific mathematician and a very ingenious one. In 1729 Euler proposed a generalization of the factorial function $n! = n(n-1)\ldots 3.2.1$ from integers to any real number. His generalization is called the gamma function $\Gamma(x)$, which is defined as [11]:

$$\Gamma(x) = \lim_{m \mapsto +\infty} \left\{ \frac{m^x m!}{x(n+1)(x+2)\ldots(x+m)} \right\} \text{......... (1).}$$

Investigators of mention include: C. Siegel, A. M. Legendre, K. F. Gauss, C. J. Malmstén, O. Schlömilch, J. P. M. Binet (1843), E. E. Kummer (1847), and G. Plana (1847). M. A. Stern (1847) proved convergence of the Stirling's series for the derivative of $\log(\Gamma(x))$. C. Hermite (1900) proved convergence of the Stirling's series for $\log(\Gamma(x+1))$, if x is a complex number [25].

During the twentieth century, the function $\log(\Gamma(x))$ was used in many research works where the gamma function was applied or investigated. The appearance of computer systems at the end of the twentieth century demanded more careful attention to the structure of branch cuts for basic mathematical functions to support the validity of the mathematical relations everywhere in the complex plane. This lead to the appearance of a special log-gamma function $\log(\Gamma(x))$, which is equivalent to the logarithm of the gamma function $\log(\Gamma(x))$ as a multivalued analytic function, except that it is conventionally defined with a different branch cut structure and principal sheet. The log-gamma function $\log(\Gamma(x))$ was introduced by J. Keiper (1990) for Mathematica [26].

The importance of the gamma function and its Euler integral stimulated some mathematicians to study the incomplete Euler integrals, which are actually equal to the indefinite integral of the expression $t^n e^{-t}$. They were introduced in an article by A. M. Legendre (1811). Later, P. Schlömilch (1871) introduced the name "incomplete gamma function" for such an integral. These functions were investigated by J. Tannery (1882), F. E. Prym (1877), and M. Lerch (1905) (who gave a series representation for the incomplete gamma function). N. Nielsen [25] and other mathematicians also had special interests in these functions, which were included in handbooks of special functions and current computer systems like Mathematica [25].
The Euler gamma function is the only function described here that is available whether or not you set, but if that flag is not set then you will be able to use the gamma function only for real arguments; to be able to use it for complex arguments you will need to set that flag. The Gamma function is initially known as an extension of the factorial function; however, it goes beyond this definition as its use extends various disciplines such as combinatorics, physics, statistics, etc [8].
Despite its uniqueness in mathematics, the gamma function unfortunately has the characteristic of possessing singularities at negative integer values and zero. It is used by the factorial function to compute non-integral values for the factorial as well. The Euler gamma function"gamma function" is an integral relationship that is defined as follows:

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx \;;\ldots\ldots\ldots\ldots\ldots (2)$$

this integral is convergent for $n \succ 0$.

The definition works for most negative values of x and even for complex values of x, but we only care about positive real values. Note that this function is automatically called implicitly whenever you do fact (arg) where arg is not an integer.
As a generalization of the Gamma function defined for a single complex variable, a new special function called a generalized Gamma function, defined for two complex variables and a positive integer, is introduced, and several important analytical properties are investigated in detail, which include regularity, asymptotic expansions and analytic continuations. Furthermore, as a function closely related to a generalized Gamma function, a generalized incomplete Gamma function, which is a generalization of the incomplete Gamma function, is also introduced, and some fundamental properties are investigated briefly [9,10].
The gamma function is finite except for non-positive integers. It goes to $+\infty$ at zero and negative even integers and to $-\infty$ at negative odd integers. The gamma function can also be uniquely extended to an analytic function on the complex plane. The only singularities are the poles on the real axis. Here's a plot of the absolute value of the gamma function over the complex plane [13].

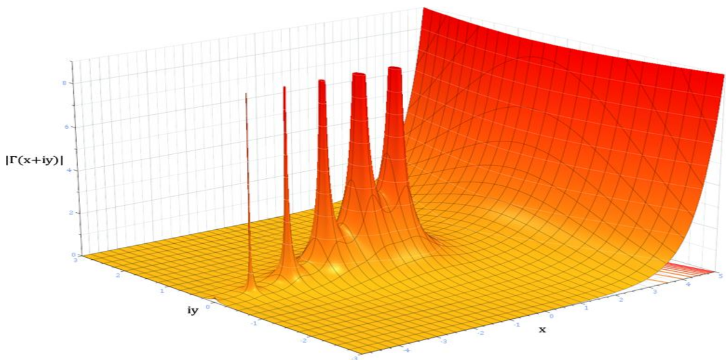

Figure 1.1: plot the absolute value of the gamma function on the complex plane.

THE KOBAYASHI'S AND AGARWAL, KALLA GAMMA TYPE FUNCTIONS

The generalized gamma function is defined (in its original) form firstly introduced by Kobayashi in 1991. Then, the appearance of this special function in analytical acoustics is briefly explained by formulating the Wiener-Hopf integral equation for a famous diffraction problem by a finite strip or a single slit [3].

We a new generalized gamma function is defined involving a parameter in the Kobayashi's (1991) function $\Gamma_r(m,n)$. The parameter r will relax the restriction on the parameter $r \succ 0$ in all cases using Kobayashi's (1991) type functions [4].
The characteristics of present formula is graphically illustrated and numerically compared with existing two formulas. Firstly, the present formula is compared with the Kobayashi's asymptotic formula (1991) with a discussion about the lower bound of available argument yielding the relative error less than 0.0001 [5]. Secondly, the present formula is compared with the Srivastava's exact formula (2005) from the viewpoint of computational accuracy and efficiency for large argument. And, finally, the author discuss about the limitation of present formula and the future work concerning a further mathematical improvement as well as the practical applications of the generalized gamma function [6].
The generalized gamma function is defined in its original form firstly introduced by Kobayashi in 1991. Despite its uniqueness, the gamma function unfortunately has the characteristic of possessing singularities at negative integer values and zero. It is used by the factorial function to compute non-integral values for the factorial as well [5]. To overcome these difficulties, Kobayashi in 1991 has introduced a new type of generalized gamma function as follows:

$$\Gamma_r(k,n) = \int_0^{\infty} x^{k-1}[x+n]^{-r} e^{-x} dx; \quad r,k,n \succ 0; \ldots\ldots(3)$$

This function is useful in many problems of diffraction theory and corrosion problems in new machines. However, this function has not been used in statistics until 1996 where Agrawal and Kalla proposed a new generalization of gamma distribution by considering a modified form of the Kobayashi's gamma function [4]. The Agarwal and Kalla gamma function"generalized gamma function" is an integral relationship that is defined as follows:

$$\Gamma_r(k,n,\lambda) = \int_0^{\infty} x^{k-1}[x+n]^{-r} e^{-\lambda x} dx; \quad r,k,n,\lambda \succ 0; \ldots\ldots(4)$$

The formula (3) proposed by the Kobayashi associated with the form (4) proposed by each Agarwal and Kalla of the parameters r,k and λ in terms of the relationship is the content of the following property (1):

Property (1): Each of the parameters r,k and λ, the following mathematical formula unrealized;

$$\Gamma_r(k,n,\lambda) = \lambda^{r-k}\ \Gamma_r(k,\lambda n)$$

Proof: Can make sure that the first parameter can not be generalized because the shape change can be adjusted and become the same formula, so:

$$\Gamma_r(k,n,\lambda) = \int_0^{\infty} x^{k-1}[x+n]^{-r} e^{-\lambda x} dx; \quad r,k,n,\lambda \succ 0.$$

Let change variable $y = \lambda x$, then $x = (y/\lambda)$ limits $\begin{matrix} x \to 0 \\ x \to +\infty \end{matrix}$, alors $\begin{matrix} y \to 0 \\ y \to \infty \end{matrix}$, then $dy = \lambda dx$, then;

$x^{k-1}[x+n]^{-r} e^{-\lambda x} = (y/\lambda)^{k-1}[(y/\lambda)+n]^{-r} e^{-y} = (\lambda)^{r-k+1}\ y^{k-1}[y+n\lambda]^{-r} e^{-y}$, and the integral;

$$\Gamma_r(k,n,\lambda) = \int_0^{\infty} (y/\lambda)^{k-1}[(y/\lambda)+n]^{-r} e^{-y}\ (dx/\lambda) = (\lambda)^{r-k} \int_0^{\infty} y^{k-1}[y+n\lambda]^{-r} e^{-y} dy$$

$$= (\lambda)^{r-k}\Gamma_r(k,n\lambda)$$

Remark: $\underset{r-k \mapsto +\infty}{lin} \Gamma_r(k,n,\lambda) = \underset{r-k \mapsto +\infty}{lin} (\lambda)^{r-k}\Gamma_r(k,n\lambda) = \begin{cases} 0, & 0 \prec \lambda \prec 1 \\ \underset{r-k \mapsto +\infty}{lin} \Gamma_r(k,n), & \lambda = 1 \\ +\infty, & \lambda \succ 1 \end{cases}$

Property (2): Each of the parameters r,k,λ and $n = 0$, the following mathematical formula unrealized;

$$\Gamma_r(k,0,\lambda) = \lambda^{r-k}\Gamma(k-r)$$

Proof: Can make sure that the first parameter can not be generalized because the shape change can be adjusted and become the same formula, so:

$$\Gamma_r(k,0,\lambda)=\int_0^\infty x^{k-1}[x]^{-r}e^{-\lambda x}dx; \quad r,k,\lambda \succ 0$$

Let change variable $y=\lambda x$, then $x=(y/\lambda)$ limits $\begin{matrix} x\to 0 \\ x\to +\infty \end{matrix}$, alors $\begin{matrix} y\to 0 \\ y\to \infty \end{matrix}$, then $dy=\lambda dx$, then $x^{k-1}[x]^{-r}e^{-\lambda x}=(y/\lambda)^{k-1}[(y/\lambda)]^{-r}e^{-y}=(\lambda)^{r-k+1}\ y^{k-r-1}e^{-y}$, and the integral;

$\Gamma_r(k,0,\lambda)=\int_0^\infty x^{k-1}[x+0]^{-r}e^{-\lambda x}dx=\int_0^\infty x^{k-r-1}e^{-\lambda x}dx=\lambda^{r-k}\int_0^\infty y^{k-r-1}e^{-y}dy=\lambda^{r-k}\Gamma(k-r)$ Is possible to cheeck the absolute value, eeal part and imaginary partof the extended general gamma function in case of $\lambda=1$ and the variable $k-r$.

Theorem (1): For all $k,n,r,\lambda \succ 0$, $k\succ r$ the improper integral function $\Gamma_r(k,n,\lambda)$ is ;

$$0\le \Gamma_r(k,n,\lambda)\prec +\infty$$

Where $\Gamma_r(k,n,\lambda)=\int_0^\infty x^{k-1}[x+n]^{-r}e^{-\lambda x}dx; \quad r,k,n,\lambda \succ 0$.

Proof: For all $k,n,r\succ 0$, we have for $x\succ 0$;

$$0\le x^{k-1}[x+n]^{-r}\le x^{k-r-1}$$

Thus, for $\lambda \succ 0$;

$$0\le \Gamma_r(k,n,\lambda)\prec \int_0^\infty x^{k-r\ -1}e^{-\lambda\, x}\,dx$$

Let change variable $y=\lambda\, x$, then $x=y/\lambda$ limits $\begin{matrix} x\to 0 \\ x\to +\infty \end{matrix}$, alors $\begin{matrix} y\to 0 \\ y\to \infty \end{matrix}$, then $dy=\lambda\, dx$, then $x^{k-r-1}e^{-\lambda x}=(y/\lambda)^{k-r-1}e^{-y}=(\lambda)^{r-k-1}y^{k-r-1}e^{-y}$, and the integral;

$$\int_0^\infty x^{k-r\ -1}e^{-\lambda\, x}\,dx=\lambda^{r-k+2}\Gamma(k-r)\prec +\infty\text{; for } k-r\succ 0.$$

Then $0\le \Gamma_r(k,n,\lambda)\prec +\infty$; for $k-r\succ 0$.

THE EXTENDED GENERALIZED GAMMA FUNCTION

Since that time many generalization of this generalized gamma function were considered by introducing new parameters. The Extended Generalized Gamma function is defined in its original form firstly introduced by Bachioua.L, in 2004 [17]. Despite its uniqueness and ubiquity in mathematics and extende of Agarwal and Kalla generalized gamma function, is an integral relationship that is defined as follows:

$$\Lambda_r(k,p,m,n,\lambda)=\int_0^\infty x^{k-1}[x^m+n]^{-r}e^{-\lambda\, x^{\,p}}dx; \quad p\,,k,p,m,n,\lambda \succ 0; \quad r\in IR \text{;...(5)}$$

The formula (5) proposed by the Bachioua associated with the form (4) proposed by each Agarwal and Kalla of the parameters $p \succ 1, k, p, m, n, r$ and λ in terms of the relationship is the content of the following property (1):

Property (2): Each of the parameters $p \succ 1, k, p, m, n, r$ and λ, the following mathematical formula unrealized [19];

$$\Lambda_r(k,1,1,n,\lambda) = \Gamma_r(k,n,\lambda)$$

Proof: The result is simple and produces direct compensation directly from the values of parameters, and therefore becomes formula the previous special case of this generalized formula property (4).

Property (4): For $r \in IR$, $p, k, p, m, n, \lambda \succ 0$. This 6-parameter function can be regarded as an extension of the Kobayashi's generalized gamma function, since;

$$\Lambda_r(k,1,1,n,1) = \int_0^{\infty} x^{k-1}\left[x+n\right]^{-r} e^{-x}\, dx = \Gamma_r(k,n); \text{ for } k, n, r > 0.$$

Also, this function is reduced to the well known gamma function when;

$$\Lambda_0(k,1,m,n,1) = \int_0^{\infty} x^{k-1} e^{-x}\, dx = \Gamma(k),\ k \succ 0$$

Theorem (2): The improper integral function is;

$$\Lambda_0(k,p,m,n,\lambda) = \frac{(\lambda)^{\frac{p-k}{p}-1}}{p}\Gamma\left(\frac{k}{p}\right); \quad \frac{k}{p} \succ 0.$$

Proof: For all $k, p, m, n, \lambda \succ 0, r = 0$,, we have for $x \succ 0$;

$$\Lambda_0(k,p,m,n,\lambda) = \int_0^{\infty} x^{k-1}\left[x^m+n\right]^{-0} e^{-\lambda x^p} dx = \int_0^{\infty} x^{k-1} e^{-\lambda x^p} dx$$

Let change variable $y = \lambda x^p$, then $x = \sqrt[p]{(y/\lambda)}$ limits $\begin{matrix} x \to 0 \\ x \to +\infty \end{matrix}$, alors $\begin{matrix} y \to 0 \\ y \to \infty \end{matrix}$, then $dy = \lambda\, p\, x^{p-1} dx$,

then $x^{k-1} e^{-\lambda x^p} = (y/\lambda)^{\frac{k-1}{p}} e^{-y} = (\lambda)^{\frac{1-k}{p}} (y)^{\frac{k-1}{p}} e^{-y}$, and the integral;

$$\begin{aligned} 0 \le \Lambda_0(k,p,m,n,\lambda) &= \int_0^{\infty} x^{k-1}\left[x^m+n\right]^{-0} e^{-\lambda x^p} dx = \int_0^{\infty} x^{k-1} e^{-\lambda x^p} dx \\ &= \frac{(\lambda)^{\frac{1-k}{p}-1}}{p} \int_0^{\infty} y^{\frac{k-1}{p}} e^{-y} (y/\lambda)^{\frac{1-p}{p}} dy \\ &= \frac{(\lambda)^{\frac{p-k}{p}-1}}{p} \int_0^{\infty} y^{\frac{k-p}{p}} e^{-y}\, dy = \frac{(\lambda)^{\frac{p-k}{p}-1}}{p} \int_0^{\infty} y^{\frac{k}{p}-1} e^{-y}\, dy \\ &= \frac{1}{p\,(\lambda)^{\frac{k}{p}}} \Gamma\left(\frac{k}{p}\right) \prec +\infty, \frac{k}{p} \succ 0. \end{aligned}$$

To investigate the properties of the function $\Lambda_r(k,p,m,n,\lambda)$, we first consider the problem of the existence of the function [16].

Theorem (3): The improper integral function $\Lambda_r(k,p,m,n,\lambda)$ is [18];

$$0 \le \Lambda_r(k,p,m,n,\lambda) \prec +\infty$$

Proof: For all $k,n,m,r \succ 0$, we have for $x \succ 0$

$$0 \le x^{k-1}\left[x^m + n\right]^{-r} \le x^{k-rm-1}$$

Thus, for $\lambda, p \succ 0$;

$$0 \le \Lambda_r(k,p,m,n,\lambda) \prec \int_0^\infty x^{k-r\,m-1} e^{-\lambda\,x^p} dx\,;$$

But,

$$\int_0^\infty x^{k-r\,m-1} e^{-\lambda\,x^p} dx = \frac{\lambda^{\frac{rm-k}{p}}}{p}\Gamma(\frac{k-rm}{p}) \prec +\infty \text{ ; For } k - rm \succ 0.$$

Hence,

$$0 \prec \Lambda_r(k,p,m,n,\lambda) \prec +\infty \text{ ; For } r \succ 0 \text{ whenever } \frac{k}{m} \text{ is large.}$$

For all $r \prec 0$; $k,n,p,m,\lambda \succ 0$, $x \succ 0$, we have, using binomial formula

$$0 \prec \left[x^m + n\right]^{-r} \le \left[x^m + n\right]^s = \sum_{i=1}^{s}\binom{s}{i} n^{s-i} x^{mi}\,,$$

where $s = -r$ when r is a negative integer, $s = [-r]+1$ when $\left[x^m + n\right] \ge 1$, and $s = [-r]$ otherwise. Thus;

$$0 \le \Lambda_r(k,p,m,n,\lambda) \le \sum_{i=1}^{s}\binom{s}{i} n^{s-i}\,\frac{\lambda^{\frac{k+i\,m}{p}}}{p}\Gamma\,(\frac{k+i\,m}{p}) \prec +\infty$$

For all $k,p,m,n,\lambda \succ 0, r = 0$, the result of Theorem (1) we have for $x \succ 0$;

$$0 \le \Lambda_0(k,p,m,n,\lambda) \prec +\infty$$

which proves the theorem?.

Results: The function $\Lambda_r(k,p,m,n,\lambda)$ can be written as follows [19];

1. $\Lambda_r(k,p,m,n,\lambda) = \dfrac{1}{p}\Lambda_r(\dfrac{k}{p},1,\dfrac{m}{p},n,\lambda)$,

2. $\Lambda_r(k,p,m,n,\lambda)=\left(\lambda^{\frac{k}{p}}p\right)^{-1}\Lambda_r(\frac{k}{p},1,\frac{m}{p},n\,\lambda^{\frac{m}{p}},1)$,

3. $\Lambda_r(k,p,m,n,\lambda)=\frac{1}{m}\Lambda_r(\frac{k}{m},\frac{p}{m},1,n,\lambda)$.

Proof: 1)-Using the substitutions $y=x^p$ and for all $k,p,m,n,\lambda\succ 0, r=0$ we have for $x\succ 0$;

$$\Lambda_r(k,p,m,n,\lambda)=\int_0^{\infty}x^{k-1}\left[x^m+n\right]^{-r}e^{-\lambda x^p}dx$$

Then $x=\sqrt[p]{(y)}$ limits $\begin{matrix}x\to 0\\ x\to+\infty\end{matrix}$, alors $\begin{matrix}y\to 0\\ y\to\infty\end{matrix}$, then $dy=p\,x^{p-1}dx$, then $x^{k-1}\left[x^m+n\right]^{-r}e^{-\lambda x^p}=y^{\frac{k-1}{p}}\left[y^{\frac{m}{p}}+n\right]^{-r}e^{-\lambda y}$, and the integral;

$$\Lambda_r(k,p,m,n,\lambda)=\int_0^{\infty}x^{k-1}\left[x^m+n\right]^{-r}e^{-\lambda x^p}dx=\frac{1}{p}\int_0^{\infty}y^{\frac{k-1}{p}}\left[y^{\frac{m}{p}}+n\right]^{-r}e^{-\lambda y}y^{\frac{1-p}{p}}dy$$

$$=\frac{1}{p}\int_0^{\infty}y^{\frac{k}{p}-1}\left[y^{\frac{m}{p}}+n\right]^{-r}e^{-\lambda y}dy=\frac{1}{p}\Lambda_r(\frac{k}{p},1,\frac{m}{p},n,\lambda).$$

2)-The second form of this theorem shows that by using the substitutions $y=\lambda x^p$ and for all $k,p,m,n,\lambda\succ 0, r=0$ we have for $x\succ 0$; hen $x=\sqrt[p]{(y/\lambda)}$ limits $\begin{matrix}x\to 0\\ x\to+\infty\end{matrix}$, alors $\begin{matrix}y\to 0\\ y\to\infty\end{matrix}$, then $dy=\lambda p\,x^{p-1}dx$, then;

$$x^{k-1}\left[x^m+n\right]^{-r}e^{-\lambda x^p}=(y/\lambda)^{\frac{k-1}{p}}\left[(y/\lambda)^{\frac{m}{p}}+n\right]^{-r}e^{-y}$$, and the integral;

$$\Lambda_r(k,p,m,n,\lambda)=\int_0^{\infty}x^{k-1}\left[x^m+n\right]^{-r}e^{-\lambda x^p}dx=\frac{1}{\lambda\,p}\int_0^{\infty}(y/\lambda)^{\frac{k-1}{p}}\left[(y/\lambda)^{\frac{m}{p}}+n\right]^{-r}e^{-y}(y/\lambda)^{\frac{1-p}{p}}dy$$

$$=\frac{\lambda^{\frac{rm+1-k+p-1}{p}-1}}{p}\int_0^{\infty}y^{\frac{k-1+1-p}{p}}\left[y^{\frac{m}{p}}+n\lambda^{\frac{m}{p}}\right]^{-r}e^{-y}dy=\frac{\lambda^{-\frac{k}{p}}}{p}\int_0^{\infty}y^{\frac{k}{p}-1}\left[y^{\frac{m}{p}}+n\lambda^{\frac{m}{p}}\right]^{-r}e^{-y}dy$$

$$=\left(\lambda^{\frac{rm-k}{p}}p\right)^{-1}\Lambda_r(\frac{k}{p},1,\frac{m}{p},n\lambda^{\frac{m}{p}},1).$$

3)-Using the substitutions $y=x^m$ and for all $k,p,m,n,\lambda\succ 0, r=0$ we have for $x\succ 0$;

Then $x = \sqrt[m]{(y)}$ limits $\begin{matrix} x \to 0 \\ x \to +\infty \end{matrix}$, alors $\begin{matrix} y \to 0 \\ y \to \infty \end{matrix}$, then $dy = m\, x^{m-1} dx$ then;

$x^{k-1}\left[x^m + n\right]^{-r} e^{-\lambda\, x^{\,p}} = y^{\frac{k-1}{m}} \left[y + n\right]^{-r} e^{-\lambda\, y^{\frac{p}{m}}}$, and the integral;

$$\begin{aligned}
\Lambda_r(k,p,m,n,\lambda) &= \int_0^\infty x^{k-1}\left[x^m + n\right]^{-r} e^{-\lambda\, x^{\,p}} dx = \frac{1}{m}\int_0^\infty y^{\frac{k-1}{m}} \left[y+n\right]^{-r} e^{-\lambda\, y^{\frac{p}{m}}} (y)^{\frac{1-m}{m}}\, dy \\
&= \frac{1}{m}\int_0^\infty y^{\frac{k-1+1-m}{m}} \left[y+n\right]^{-r} e^{-\lambda\, y^{\frac{p}{m}}} dy = \frac{1}{m}\int_0^\infty y^{\frac{k}{m}-1} \left[y+n\right]^{-r} e^{-\lambda\, y^{\frac{p}{m}}} dy \\
&= \frac{1}{m}\Lambda_r\left(\frac{k}{m}, \frac{p}{m}, 1, n, \lambda\right).
\end{aligned}$$

For the values of parameters (k,m,n,r) in $\Lambda_r(k,p,m,n,\lambda)$ are essential, while the others (p,λ) can be regarded as index parameters. Also, when $p \to +\infty$, or $m \to +\infty$, we have $\Lambda_r(k,p,m,n,\lambda) \to 0$ [18].

Theorem (4): The improper integral function $\Lambda_r(k,p,m,n,\lambda)$ is satisfies the following recurrence relations;

1)- $\Lambda_r(k,p,m,n,\lambda) = \dfrac{1}{\lambda\, p}\left[\left(\dfrac{k}{p} - 1\right)\Lambda_r\left(\dfrac{k}{p} - 1, 1, \dfrac{m}{p}, n, \lambda\right) - \dfrac{r\, m}{p}\Lambda_{r+1}\left(\dfrac{k+m}{p} - 1, 1, \dfrac{m}{p}, n, \lambda\right)\right]$

2)- $\Lambda_r(k,p,m,n,\lambda) = \dfrac{\lambda^{\frac{r\, m - k}{p}}}{p}\left[\left(\dfrac{k}{p} - 1\right)\Lambda_r\left(\dfrac{k}{p} - 1, 1, \dfrac{m}{p}, n\, \lambda^{\frac{m}{p}}, 1\right) - \dfrac{r\, m}{p}\Lambda_r\left(\dfrac{k+m}{p} - 1, 1, \dfrac{m}{p}, n\lambda^{\frac{m}{p}}, 1\right)\right]$

Proof: This follows by applying integration by parts to the form (1) and (2) of $\Lambda_r(k,p,m,n,\lambda)$ given in theorem (5), respectively, then:

$$\Lambda_r(k,p,m,n,\lambda) = \frac{1}{p}\Lambda_r\left(\frac{k}{p}, 1, \frac{m}{p}, n, \lambda\right) = \frac{1}{p}\int_0^\infty y^{\frac{k}{p}-1}\left[y^{\frac{m}{p}} + n\right]^{-r} e^{-\lambda y} dy$$

$$= \frac{1}{\lambda\, p}\left[\left(\frac{k}{p} - 1\right)\Lambda_r\left(\frac{k}{p} - 1, 1, \frac{m}{p}, n, \lambda\right) - \frac{r\, m}{p}\Lambda_{r+1}\left(\frac{k+m}{p} - 1, 1, \frac{m}{p}, n, \lambda\right)\right]$$

$$\Lambda_r(k,p,m,n,\lambda) = \frac{\lambda^{-\frac{k}{p}}}{p}\int_0^\infty y^{\frac{k}{p}-1}\left[y^{\frac{m}{p}} + n\lambda^{\frac{m}{p}}\right]^{-r} e^{-y} dy$$

$$= \frac{\lambda^{\frac{r\, m-k}{p}}}{p}\left[\left(\frac{k}{p} - 1\right)\Lambda_r\left(\frac{k}{p} - 1, 1, \frac{m}{p}, n\, \lambda^{\frac{m}{p}}, 1\right) - \frac{r\, m}{p}\Lambda_r\left(\frac{k+m}{p} - 1, 1, \frac{m}{p}, n\lambda^{\frac{m}{p}}, 1\right)\right]$$

Theorem (5): When $p = m$, we have;

$$\Lambda_r(k,p,m,n,\lambda) = \left(\frac{\lambda^{\frac{r\,m-k}{m}}}{m}\right)\Lambda_r(\frac{k}{m}-1,1,1,n\,\lambda,1) = \left(\frac{\lambda^{\frac{r\,m-k}{m}}}{m}\right)\Gamma\left(\frac{k}{m}-r\right);\text{ for } \frac{k}{m}\succ r \text{ and for small}$$

$n\,\lambda$.

Proof: From the result theorem (3), then;

$$\Lambda_r(k,p,m,n,\lambda) = \frac{\lambda^{\frac{r\,m-k}{m}}}{m}\left[\left(\frac{k}{m}-1\right)\Lambda_r(\frac{k}{m}-1,1,\frac{m}{m},n\,\lambda^{\frac{m}{m}},1) - \frac{r\,m}{m}\Lambda_r(\frac{k+m}{m}-1,1,\frac{m}{m},n\lambda^{\frac{m}{m}},1)\right]$$

$$= \left(\frac{\lambda^{\frac{r\,m-k}{m}}}{m}\right)\Lambda_r(\frac{k}{m}-1,1,1,n\,\lambda,1) = \left(\frac{\lambda^{\frac{r\,m-k}{m}}}{m}\right)\Gamma\left(\frac{k}{m}-r\right).$$

and the proof follows.

Theorem (6): When $n = 0$, we have;

$$\Lambda_r(k,p,m,0,\lambda) = \left(\frac{\lambda^{\frac{r\,m-k}{p}}}{p}\right)\Gamma\left(\frac{k}{m}-r\right).;\text{ for } \frac{k}{m}\succ r.$$

Proof: Using the transformation $y = \lambda x^p$ and for all $k,p,m,n,\lambda \succ 0, r = 0$ we have for $x \succ 0$; hen $x = \sqrt[p]{(y/\lambda)}$ limits $\begin{matrix} x \to 0 \\ x \to +\infty \end{matrix}$, alors $\begin{matrix} y \to 0 \\ y \to \infty \end{matrix}$, then $dy = \lambda p\, x^{p-1}dx$, then; $x^{k-1}\left[x^m + n\right]^{-r} e^{-\lambda\, x^p} = (y/\lambda)^{\frac{k-1}{p}}\left[(y/\lambda)^{\frac{m}{p}} + n\right]^{-r} e^{-y}$, and for $n = 0$ the integral;

$$\Lambda_r(k,p,m,0,\lambda) = \int_0^\infty x^{k-1}\left[x^m + 0\right]^{-r} e^{-\lambda\, x^p}dx = \frac{1}{\lambda\, p}\int_0^\infty (y/\lambda)^{\frac{k-1}{p}}\left[(y/\lambda)^{\frac{m}{p}}\right]^{-r} e^{-y}(y/\lambda)^{\frac{1-p}{p}}dy$$

$$= \frac{\lambda^{\frac{r\,m-k}{p}}}{p}\int_0^\infty y^{\frac{k-r\,m}{m}-1}e^{-y}dy == \left(\frac{\lambda^{\frac{r\,m-k}{p}}}{p}\right)\Gamma\left(\frac{k}{m}-r\right).$$

and the proof follows.

Another feature of this function $\Lambda_r(k,p,m,n,\lambda)$ it is produce several important new sub-types of the extended generalized gamma function, simply by reducing the number of parameters of $\Lambda_r(k,p,m,n,\lambda)$ to less than 6-parameters by assigning proper values to the eliminated parameter [19].

In the next section, we shall define a new type of distribution based on the function $\Lambda_r(k,p,m,n,\lambda)$ and study many of its properties. The extension, which occurred in the dissemination of the gamma function allows the definition of functions over an extended version of the generalized gamma, which is known in many different cases are similar to each other in special cases, vary from each other in other cases, the form displays under some cases for clarification.

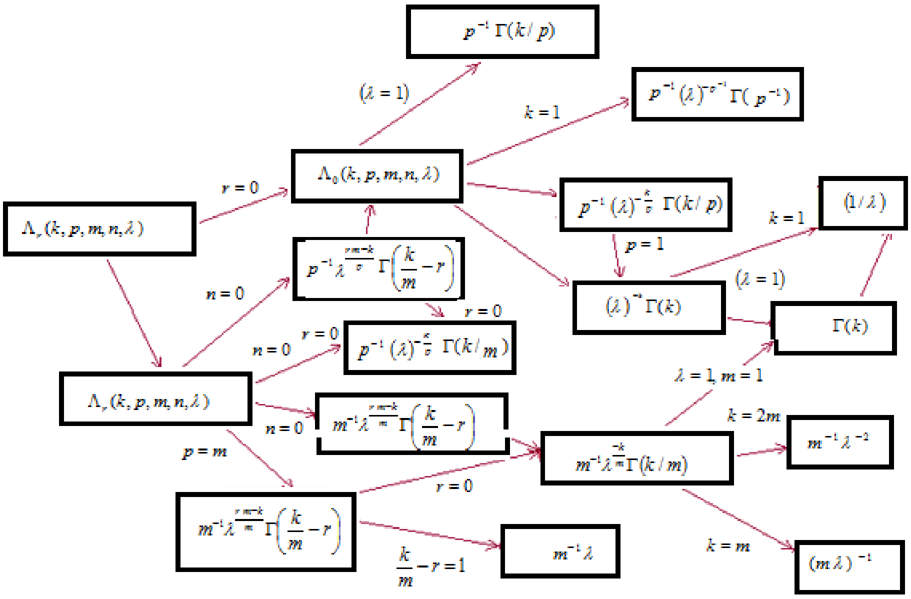

Figure 3.1 :The representing of the Extended Generalized Gamma set of functions.

THE EXTENDED GENERALIZED GAMMA MODEL FUNCTION

Because of the role of displacement parameters and positioning, shape, and the Commission, the researcher developed a general formula using a public function in general is as the following integral;

$$\Lambda_r(k,p,m,n,\lambda)=\int_a^b \alpha(x)^{k-1}\left[\alpha(x)^m+n\right]^{-r}e^{-\lambda\,\alpha(x)^p}\;\alpha(x)'\,dx;\quad p,k,p,m,n,\lambda\succ 0;\, r\in IR\ldots(5)$$

The $\alpha(.)$ function is required to be derived are defined in the field and check the condition of the values $[a,b]$ of the limits of integration, so that:

$$\begin{aligned}&\alpha(.):\quad [a,b]\rightarrow IR^{+}\\ &\quad x\mapsto \alpha(x);\quad with\ condition\ \alpha(a)=0,\ \alpha(b)=+\infty\end{aligned}$$

The formula (4) extended by proposed function associated with the form (3) proposed by Bachioua of the function $\alpha(.)$ in terms of the relationship is the content of the following property (4):

Property (5): Each of the function $\alpha(.)$, the following mathematical formula unrealized;

$$\begin{aligned}\Lambda_r(k,p,m,n,\lambda)&=\int_a^b \alpha(x)^{k-1}\left[\alpha(x)^m+n\right]^{-r}e^{-\lambda\,\alpha(x)^p}\;\alpha(x)'\,dx;\quad p,k,p,m,n,\lambda\succ 0;\, r\in IR\\ &=\int_0^\infty x^{k-1}\left[x^m+n\right]^{-r}e^{-\lambda x^p}\;\;dx;\quad p,k,p,m,n,\lambda\succ 0;\, r\in IR\end{aligned}$$

Proof: Can make sure that the first parameter can not be generalized because the shape change can be adjusted and become the same formula, so:

$\Lambda_r(k,p,m,n,\lambda)=\int_a^b \alpha(x)^{k-1}\left[\alpha(x)^m+n\right]^{-r} e^{-\lambda\,\alpha(x)^p}\ \alpha\ (x)'\,dx;\quad p\ ,k,p,m,n,\lambda \succ 0; r\in IR$ Let change variable $y=\alpha\ (x)$, then $d\ \alpha(x)=\alpha(x)'dx$ limits $\begin{matrix} x\to a \\ x\to b\end{matrix}$, alors $\begin{matrix} y\to 0 \\ y\to \infty\end{matrix}$, then, then

$$\alpha(x)^{k-1}\left[\alpha(x)^m+n\right]^{-r} e^{-\lambda\,\alpha(x)^p} = y^{k-1}\left[y^m+n\right]^{-r} e^{-\lambda\, y^p}\ \alpha(x)$$

$$\begin{aligned}\Lambda_r(k,p,m,n,\lambda) &= \int_a^b \alpha(x)^{k-1}\left[\alpha(x)^m+n\right]^{-r} e^{-\lambda\,\alpha(x)^p}\ \alpha\ (x)'\,dx \\ &= \int_0^\infty x^{k-1}\left[x^m+n\right]^{-r} e^{-\lambda\, x^p}\ \ dx;\quad p\ ,k,p,m,n,\lambda \succ 0; r\in IR\end{aligned}$$

Remark (1): For $\alpha\ (x)=\left(\dfrac{x-\eta}{\sigma}\right)$, then the extende generalized gamma function is as the following integral:

$$\Lambda_r(k,p,m,n,\lambda,\eta,\sigma)=\frac{1}{\sigma}\int_a^b \left(\frac{x-\eta}{\sigma}\right)^{k-1}\left[\left(\frac{x-\eta}{\sigma}\right)^m+n\right]^{-r} e^{-\lambda\left(\frac{x-\eta}{\sigma}\right)^p}\ \alpha\ (x)'\,dx; \text{..... (6)}$$

Where $p\ ,k,p,m,n,\lambda \succ 0; r,\eta\in IR, \sigma\in IR^*$,

The proposed formula in the formula (5) formulated in terms of eight parameters to help researchers in the removal and the withdrawal of the curve in the possible directions which paves the way for the definition of distributions formulated in proportion with the need to track the change of basic data, and is this form of the proposal, the case of lying to extend Unable to expand the applications targeted.

Remark (2): The proposed formula for the function $\alpha\ (.)$ and the basic conditions can be function as the following:

$$\alpha\ (x)=\left(\frac{x-a}{b-x}\right);\quad x\in(a,b)$$

This formula compatible with the basic conditions and allow for expansion of the field matches the generalized beta function as a special case of the extended gamma function, and in view of the proposed formula for alpha function as follows [20]:

$$\alpha\ (x)=(x-a)(b-x)^{-1}\ ;\quad x\in(a,b)$$

Let change variable $\alpha\ (x)=(x-a)(b-x)^{-1}$, then $d\ \alpha(x)=\dfrac{b-a}{(b-x)^2}dx$ limits $\begin{matrix} x\to a \\ x\to b\end{matrix}$, alors $\begin{matrix} y\to 0 \\ y\to \infty\end{matrix}$, then;

$$\alpha(x)^{k-1}\left[\alpha(x)^m+n\right]^{-r} e^{-\lambda\,\alpha(x)^p} = \left((x-a)(b-x)^{-1}\right)^{k-1}\left[\left((x-a)(b-x)^{-1}\right)^m+n\right]^{-r} e^{-\lambda\left((x-a)(b-x)^{-1}\right)^p}$$

$$\Lambda_r(k,p,m,n,\lambda)=\int_a^{+b}\alpha(x)^{k-1}\left[\alpha(x)^m+n\ \right]^{-r}e^{-\lambda\,\alpha(x)^p}\ \alpha\ (x)'\,dx$$

$$=\int_0^{\infty}\left((x-a)(b-x)^{-1}\right)^{k-1}\left[\left((x-a)(b-x)^{-1}\right)^m+n\ \right]^{-r}e^{-\lambda\left((x-a)(b-x)^{-1}\right)^p}\ \frac{b-a}{(b-x)^2}dx\quad ;$$

$$=(b-a)\int_0^{\infty}\frac{(x-a)^{k-1}}{(b-x)^{k+1}}\left[\frac{(x-a)^m+n\,(b-x)^m}{(b-x)^m}\right]^{-r}e^{-\lambda\left((x-a)(b-x)^{-1}\right)^p}dx$$

$$=(b-a)\int_0^{\infty}(x-a)^{k-1}(b-x)^{r\,m-k-1}\left[(x-a)^m+n\,(b-x)^m\ \right]^{-r}e^{-\lambda\left((x-a)(b-x)^{-1}\right)^p}dx$$

Result: Recent modifications that allow the researcher defines a new model function with eight parameters, and smooth allows finding a common formula with the function of the generalized beta function previously proposed, and this function is in the form:

$$\Lambda_r(k,p,m,n,\lambda,a,b)=(b-a)\int_0^{\infty}(x-a)^{k-1}(b-x)^{r\,m-k-1}\left[(x-a)^m+n\,(b-x)^m\right]^{-r}e^{-\lambda\left((x-a)(b-x)^{-1}\right)^p}dx\quad ;$$

where $p\ ,k,p,m,n,\lambda\succ 0;\ r,a\in IR,\ b\in IR^*,a\prec b$.

APPLICATIONS

In just the past thirty years several new special functions and applications have been discovered. This treatise presents an overview of the area of special functions, is one of the most important functions in analysis and its applications, the history and the development of this function in Mixture Model and the Estimation of Hazard Rate and Reliability General Mixture Gamma Distribution Model, are described in detail in a paper [16,17].

The Extended Generalized Gamma Function, which is the derivative of this function, is also commonly seen, and was discovered in theoretical physics. It has other applications as well, for example it's extensively in probability, and for example it's extensively in probability theory for reasons quite unrelated to its factoral connection, a mathematician recommended somebody as being very bright, very knowledgeable, and interested in applications[, 28, 29, 32].

The Extended Generalized Gamma Function has important applications in probability theory, combinatory and most, if not all, areas of physics for same typical applications for displays as visual stimulators and factors and its application to Stirling's formula. One of the principal applications of these functions was in the compact expression of approximations to physical problems for which explicit analytical solutions could not be found. Formal grammar, while interesting for its own sake, is rarely useful to those who use natural language to communicate.

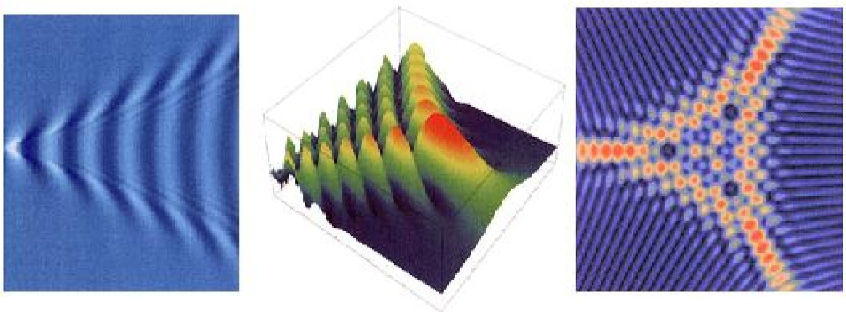

Figure 5.1: A graph of EGGF function in the hierarchy of diffraction catastrophes.

Arguing by analogy, I wonder if that is why the formal classifications of special functions have not proved very useful in applications, the simplest special function in the hierarchy of diffraction catastrophes, a cross section of the elliptic umbilicus, a member of the hierarchy of diffraction catastrophes. The cusp, a member of the hierarchy of diffraction catastrophes , just as new words come into the language, so the set of special functions increases. The increase is driven by more sophisticated applications, and by new technology that enables more functions to be depicted in forms that can be readily assimilated.

Its possible application in reliability theory, to study displacement phenomenon of the corrosion problem in a new machine or metal fatigue, and mainly introduced in order to extend the scope of ordinary and extend gamma function. There are, however, no significant applications where the gamma function by itself constitutes the essence of the solution; the exploitation of special functions provides a powerful method for solving definite integrals, in particular those encountered by practical engineering applications [24, 27].

Recent articles demonstrated that the generalized gamma function models and extended function model equations for extended generalized gamma models provide flexible approaches to deal with a variety of data problems encountered in expenditure estimation. To date there have been few empirical applications of these models to expenditures for extended generalized gamma type functions are obtained for objective in probability density functions and applications in statistics. We justify the importance of this class of models in practice using a set of real time series data. It is shown that this approach leads to a significant improvement in the quality of forecasts of correlated data and opens a new direction of research for statistical quality control. A good solution to this is the extended generalized gamma function [30].

In future, we investigate the application of the extended generalized gamma function, their application in studying robustness and model-dependence in lifetime in data space by extending the family of models.

REFERENCES

1. Abramowitz Milton and Irene A. Stegun, eds., (1972)."Handbook of Mathematical Functions with Formulas", Graphs, and Mathematical Tables. New York: Dover.
2. Abramowitz, M. and Stegun, I.A. (1964)."Handbook of Mathematical Functions with Formulas, Graphs", and Mathematical Tables, Dover, New York.
3. Agarwal, S.K. and Al- Saleh, Jamal., (2001)."Generalized gamma type distribution and its hazard rate function", Communication In Statistics: Theory and Method, 30(2), 309-318.
4. Agarwal, S.K. and Kalla, S.L, (1996)."A generalized gamma distribution and its applications in relativity". Comm. in Statist. Theory and Methods, Vol. 25, 201-210.
5. Agarwal, S.K. and Kalla, S.L., (1996)."A generalized gamma distribution and its application in reliability", Communication In Statistics: Theory and Method.
6. Al- Saleh, Jamal and Agarwal, S.K., (2007)."Finite Mixture of Gamma Distribution: A Conjugate Prior", Computational Statistics and Data Analysis, 51(9), 4369-78.
7. Al-Musallam, F. and Kalla, S. (1998)."Further results on a generalized gamma function occurring in diffraction theory", Integral transforms, special functions, 7, 175-190.
8. Al-Musallam, F. and Kalla, S.L., (1997)."Asymptotic expansions for generalized gamma and incomplete gamma functions", Applied Analysis, Vol. 66, 173-187.
9. Al-Musallam, F. and Kalla, S.L., (1998) ."Further results on a generalized gamma function ocurring in diffraction theory", Integral Transforms and Special Function, Vol. 7, 175-190.
10. Alzer, H., (2000) ."A mean-value inequality for the gamma function", Applied Math. Letters, 13, 111-114.
11. Alzer, H., (1994)."On some inequalities involving $(n!)^{1/n}$", II, Period. Math. Hung., 28(3), 229-233.
12. Alzer, H., (2000)."A power mean inequality for the gamma function", Monatsh. Math,131, 179-188.
13. Alzer, H. and Ruscheweyh, S ., (2003)."A subadditive property of the gamma function", J. Math. Anal. Appl., 285, 564-577.
14. Alzer, H., (1995) ."Note on an inequality involving $(n!)^{1/n}$", Acta Math. Univ. Comen. New Ser., 64, 283-285.
15. Alzer, H., (2000) ."Inequalities for the gamma function", Proc. Amer. Math. Soc., 128,141-147.
16. Bachioua, Lahcene, (2008) ."Extended Generalized Type Mixture Model and the Estimation of Hazard Rate", International Conference on Recent Trends in Mathematical Sciences (ICRMS2008), Department of Mathematics, College of Science , University of Bahrain, November 10 th – 12 th Bahrain.
17. Bachioua, Lahcene., (2004) ."On Extended and Reliability General Mixture Gamma Distribution Model", A Dissertation Submitted to The College of Science / University of Baghdad in Partial Fulfillment of The Requirements for The Degree of Doctor of Philosophy (Ph.D) of Science in Mathematics, University of Baghdad, Iraq.
18. Bachioua, Lahcene., (2006)."On Generalized Gamma Distribution Function", First conference in Mathematics, Department of mathematics, college of Applied science and Mathematics Zarqa Private University Amman; 18-20 April. Jordan.
19. Bachioua, Lahcene., (2009)."On Extended Generalized Gamma Distribution", International Journal of Applied Mathematics & Statistics, December, Volume 15, Number D09, CESER Publications, Roorkee-247667, INDIA.
20. Bachioua, Lahcene., (2011) ."On Extended Generalized Gamma Distribution Function", Third conference in Mathematics, Department of mathematics, college of Applied science and Mathematics Zarqa Private University ; 18-20 April. Jordan.

21. Bank, S.B., (1977)."Some results on the gamma function and other hypertranscendental functions", Proc. Royal Soc. Edinb. Sect. A 79(1977/78), no.3-4, 335-341.
22. Dragomir, S.S., Agarwal, R.P. and Barnett, N.S., (1999) ."Inequalities for beta and gamma functions via some classical and new inequalities", RGMIA Research Report Collection (Australia), 2, no.3,283-333.
23. Galué, L., (2002)."Differintegrals of Wright's generalized Hypergeometric Function", International Journal of Applied Mathematics, Vol. 30 (3), 255-267
24. Ghitany, M.E. (1998) ."On a recent generalization of gamma distribution", Communications in Statistics—Series A: Theory and Methods, 27, 223–233.
25. Gronwall, T.H. ., (1919)."The gamma function in the integral calculus", Annals of Math., 20, 35-124.
26. Haruki, H., (1986) ."A new characterization of Euler's gamma function by a functional equation", Aeq. Math., 31, no.2-3, 173-183.
27. Hung W.L. and J.W. Wu, (1999)."Some Properties of the Extended Generalized Logistic-Gamma Distribution with Applications", International Journal of Information and Management Sciences 10(4), 41-58.
28. Kalla, S.L., Al-Saqabi, B.N. and Khajah, H., (2001")."A unified form of gamma type distributions", Applied Mathematics and Computation, V 118, (2-3), 175-187.
29. Stacy E. W., (1962) ."A generalization of the gamma distribution", Annals of Mathematical Statistics, vol. 33, pp. 1187–1192.
30. Thakur, D.S., (1996) ."Transcendence of gamma values for Fq[T]", Annals of Math., (2)144, no.1, 181-188
31. Virchenko, N., Kalla, S.L. and Al-Zamel, A., (2001)."Some results on a generalized hypergeometric function", Integral Transforms and Special Functions, 12 (1), 89-100
32. Wu J.W., W.L. Hung and H.M. Lee, (2000)."Some Moments and Limit Behaviors of the Generalized Logistic Distribution with Applications", Proc. Natl. Sci. Counc.

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