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Using Matrix to Solving the Probabilistic Inventory Models (Demand Model)

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ABSTRACT
An inventory can be defined as a stock of goods which is held for the purpose of future production or sales. An inventory problem exists if the amount of the goods in stock (i.e., inventory) is subject to control and if there is at least one cost that decreases as inventory increases. Inventory models are of two types, one is inventory models with deterministic demand and other is inventory models with probabilistic demand.
Aim of the paper is to investigate a new approach to solve single period discrete probabilistic inventory models. This method proposes momentous advantages over similar methods.
Keywords: Probabilistic Inventory Model, Deterministic Demand Model, Pay-off Matrix.

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INTRODUCTION

Inventories occupy the most strategic position in the structure of working capital of most business enterprises. It constitutes the largest component of current asset in most business enterprises. In the sphere of working capital, the efficient control of inventory has passed the most serious problem to the cement mills because about two-third of the current assets of mills are blocked in inventories. The turnover of working capital is largely governed by the turnover of inventory. It is therefore quite natural that inventory which helps in maximize profit occupies the most significant place among current assets.

Meaning and Definition of Inventory: In dictionary meaning of inventory is a "detailed list of goods, furniture etc". Many understand the word inventory, as a stock of goods, but the generally accepted meaning of the word 'goods' in the accounting language, is the stock of finished goods only [1,2]. In a manufacturing organization, however, in addition to the stock of finished goods, there will be stock of partly finished goods, raw materials and stores. The collective name of these entire items is 'Inventory' [3,4].

The term 'inventory' refers to the stockpile of production a firm is offering for sale and the components that make up the production [5]. The inventory means aggregate of those items of tangible personal property which

(i) Are held for sale in ordinary course of business.
(ii) Are in process of production for such sales.
(iii) They are to be currently consumed in the production of goods or services to be available for sale.

Inventories are expandable physical articles held for resale for use in manufacturing a production or for consumption in carrying on business activity such as merchandise, goods purchased by the business which are ready for sale. It is the inventory of the trader who dies not manufacture it.

Finished Goods: Goods being manufactured for sale by the business which are ready for sale.

Materials: Articles such as raw materials, semi-finished goods or finished parts, which the business plans to incorporate physically into the finished production.

Supplies: Article, which will be consumed by the business in its operation but will not physically as they are a part of the production. The short inventory may be defined as the materials, which are either saleable in the market or usable directly or indirectly in the manufacturing process [6,7,8]. It also includes the items which are ready for making finished goods in some other process or by comparing them either by the concern itself and/or by outside parties. In other words, the term inventory means the material having any one of the following characteristics. It may be

1. Saleable in the market,
2. Directly saleable in the manufacturing process of the business,

3. Usable directly in the manufacturing process of the undertaking, and
4. Ready to send to the outside parties for making usable and saleable productions out of it.

In the present study raw materials, stores and spare parts, finished goods and work-in-process have been included inventories. Firm also manufactures inventory to supplies.

Supplies included office and plant cleaning materials (soap, brooms etc. oil, fuel, light bulbs and the likes). These materials do not directly enter into the production process, but are necessary for production process. Inventory constitutes the most significant part of current assets of a large majority of companies in India. For example, on an average inventories are more than 57 per cent of current assets in public limited companies and about 60.5per cent in government companies in India. Therefore it is absolutely imperative to manage inventories efficiently and effectively in order to avoid unnecessary investment in them. An undertaking neglecting the management of inventories will be jeopardizing its long run profitability and may fail ultimately. It is possible for a company to reduce its level of inventories to a considerable degree *e.g.* 10 to 20 per cent without any adverse effect on production and sales.

OBJECTIVES OF INVENTORY MANAGEMENT

The primary objectives of inventory management are:

(i) To minimize the possibility of disruption in the production schedule of a firm for want of raw material, stock and spares.

(ii) To keep down capital investment in inventories.

So it is essential to have necessary inventories. Excessive inventory is an idle resource of a concern. The concern should always avoid this situation. The investment in inventories should be just sufficient in the optimum level. The major dangers of excessive inventories are:

(i) The unnecessary tie up of the firm's funds and loss of profit.

(ii) Excessive carrying cost, and

(iii) The risk of liquidity.

The excessive level of inventories consumes the funds of business, which cannot be used for any other purpose and thus involves an opportunity cost. The carrying cost, such as the cost of shortage, handling insurance, recording and inspection, are also increased in proportion to the volume of inventories. This cost will impair the concern profitability further.

Problems faced by management:

(i) To maintain a large size inventories for efficient and smooth production and sales operation.

(ii) To maintain only a minimum possible inventory because of inventory holding cost and opportunity cost of funds invested in inventory.

(iii) Control investment in inventories and keep it at the optimum level.

Inventory management, therefore, should strike a balance between too much inventory and too little inventory. The efficient management and effective control of inventories help in achieving better operational results and reducing investment in working capital [9-11]. It has a significant influence on the profitability of a concern.

MATHEMATICAL FORMULATION AND ANALYSIS

Probabilistic Inventory Models:

These are the models where the demand is probabilistic or random variable. Following either a discrete or continuous probability distribution. There would be single period model and multi period models. Single period model is discussed in this paper.

Single period discrete probabilistic demand model:

These models deal with the inventory situation of the item requiring one time purchase only, such as news papers, perishable goods, spare parts and seasonal goods. These items should be stocked at the beginning of a given time period. The demand is unknown, but the probability distribution of demand is given. In these cases purchases are made only once, i.e. no reordering is possible during the period, if more units are required than stocked. Thus, the lead time factor and ordering costs are least important in these models. In single period models, the problem can be studied using marginally (or incremental) or payoff matrix method. The decision procedure consists of a sequence of steps. In such cases, there are two types of costs involved, namely (a) over-stocking cost and (b) under-stocking cost representing opportunity losses incurred when the number of units stocked is not exactly equal to the number of units actually demanded.

The objective is to determine the initial inventory level which will optimize the expected return, taking into account factors such as unit cost, carrying or holding cost, selling price, shortage cost and salvages value.

The models are discussed using the following notations:

D = number of unite of the item demanded (a random variable)

Q = the number of units of the item stocked

C = unit cost price

C_h = the unit carrying cost for the entire period

C_s = the shortage cost

P = the unit selling price

S = the salvage value

C_1 = over-stocking cost or over-ordering cost, (i.e. an opportunity loss associated with each unit left unsold)

$= C + C_h - V$

C_2 = under-stocking cost or under-ordering cost, (i.e. an opportunity loss due to not meeting the demand),

$= P - C + C_h / 2 + C_S$

Single period probabilistic demand model can be discrete or continuous. The discrete problem can be solved through incremental analysis method and payoff matrix method, as discussed below

Incremental Analysis Method for Single Period Discrete Probabilistic Inventory Model:

The cost equation may be developed as following:

(a) If only D units are consumed, for any quantity in the stock Q, for the cost associated with Q units in stock for the specified period of time, is either:

$(Q - D)C_1$, when $D \le Q$ or $(Q - D)C_2$, when $D > Q$. .... (1)

(b) Since, the demand D is random variable; its probability distribution is known. If p(D) denotes the probability of demand (D units), such that total probability is 1, i.e.

$$p(0) + p(1) + ... + p(\mathrm{D}) + ... = \sum_{D=0}^{\infty} p(\mathrm{D}) = 1. \qquad .... (2)$$

(c) The sum of expected under stocking cost and overstocking cost would be the total expected cost, say F(Q), which is given by

$$F(Q) = C_1 \sum_{D=0}^{Q} (Q - D) p(\mathrm{D}) + C_2 \sum_{D=Q+1}^{\infty} (D - Q) p(\mathrm{D}). \qquad .... (3)$$

(d) If quantity stocked is optimal, than Q^*, the total expected cost F(Q^*) will be minimum. Thus, if one unit more or less than the optimal quantity is stocked, the total expected cost will be higher than optimal.

For minimum of F (Q^*), the condition

$\Delta F(Q^* - 1) < 0 < \Delta F(Q^*)$ must be satisfied.

The equation (3) can be differenced under the summation sign, for $D = Q^* + 1$

$$\mathrm{C}_1[(Q^* + 1) - \mathrm{D}]\,\mathrm{p(D)} = \mathrm{C}_2[D - (Q^* + 1)\,\mathrm{p(D)}]. \qquad .... (4)$$

This is obviously satisfied here. Now taking first difference of equation (3) , following is obtained

$$\mathrm{F(Q^*)} = \mathrm{C}_1 \sum_{D=0}^{\mathrm{Q}^*} [(Q^* + 1) - (\mathrm{D}) - (Q^* - \mathrm{D})]\,\mathrm{p(D)} + \mathrm{C}_2 \sum_{D=Q^*+1}^{\infty} [\mathrm{D} - (Q^* + 1) - (\mathrm{D} - Q^*)]\,\mathrm{p(D)}$$

$$\mathrm{F(Q^*)} = \mathrm{C}_1 \sum_{D=0}^{\mathrm{Q}^*} \mathrm{p(D)} - \mathrm{C}_2 \left[\sum_{D=Q^*+1}^{\infty} \mathrm{p(D)} - \sum_{D=0}^{\mathrm{Q}^*} \mathrm{p(D)} \right]$$

$$F(Q^*) = (C_1 + C_2)\sum_{D=0}^{Q^*} p(D) - C_2 \text{ (Because } \sum_{D=0}^{\infty} = 1\text{)} \qquad \text{......... (5)}$$

From equation (4), $\Delta F(Q^*) > 0$

$$(C_1 + C_2)\sum_{D=0}^{Q^*} p(D) - C_2 > 0$$

or $\sum_{D=0}^{Q^*} p(D) > \frac{C_2}{(C_1 + C_2)}$. .... (6)

Thus, the optimum value of stock level Q^* can be obtained by the relationship

$$\sum_{D=0}^{Q^*-1} p(D) < \frac{C_2}{(C_1 + C)} < \sum_{D=0}^{Q^*} p(D) . \qquad \text{......... (7)}$$

For practical application of equation (7) , the three step procedure is as follows:

Step-I. A table showing p(D), the probability and the cumulative probability $p(D \le Q)$ for each reasonable value of D is prepared.

Step-II. The ratio $\frac{C_2}{(C_1 + C_2)}$ known as service level is computed.

Step-III. The value of Q satisfying the equation (7) is obtained.

Example-1. A trader stocks woollen sweaters at the beginning of winter and cannot re-order. The item costs him Rs. 50 each and he sells at Rs. 100. For each sweater that cannot be met on demand, the trader loses goodwill of Rs. 30. Each unsold piece will have salvage value of Rs. 20. Holding cost during the period is 10 persent of the item cost. The probability of demand is as follows:

Unit stocked (in hundreds):	4	6	8	10	12
Demand probability $p(D \le Q)$:	0.30	0.20	0.30	0.15	0.05

Compute the optimal number of item to be stocked. If a stock level is to be maintained 10 units, find the values of under stocking cost (C_2).

Solution:

Method-I: The data regarding demand distribution given in problem is tabulated in Table 1:

Unit stocked (in hundreds)	4	6	8	10	12
Demand probability p(D=Q)	0.30	0.20	0.30	0.15	0.05
Cumulative probability $p(D \le Q)$	0.30	0.50	0.80	0.95	1.00

Table-1: Probability Distribution of demand of woollen sweaters

In the problem, P=100, C=50, C_h =0.10×50=5.0, C_s =30, S=20

Therefore, C_1 =C+ C_h -S=50+5.0-20=35

$$C_2 = P - C - \frac{C_h}{2} + C_s = 100 - 50 - \frac{5.0}{2} + 30 = 77.5$$

Thus, $\frac{C_2}{(C_1 + C_2)} = \frac{77.5}{(35 + 77.5)}$ =0.69.

Observing the table 1, this ratio lies between cumulative probability of 0.50 and 0.80 which in turn, reflect the values of Q as 6 and 8. That is,

$p(D \le 6)$ =0.50<0.69<0.80= $p(D \le 8)$.

Therefore, the optimum number of units to stock is 4 units.

Cost of under stocking can be calculated as follows:

From the problem, following inequality can be found

$$p(D \le 8) \le \frac{C_2}{(35 + C_2)} \le p(D \le 10) \text{ or } 0.80 \le \frac{C_2}{(35 + C_2)} \le 0.95.$$

The minimum value of C_2 is determined by letting

$\frac{C_2}{(35+C_2)}$ =0.80 or $C_2=\frac{(0.80)(35)}{(1-0.80)}$ = Rs. 140.

The maximum value of C_2 is determined by letting

$\frac{C_2}{(35+C_2)}$ =0.95 or $C_2=\frac{(0.95)(35)}{(1-0.95)}$ = Rs. 665

Therefore, 140 ≤ C_2 ≤665.

Method-II: Matrix Method

This method can be explained by considering again the above example. The trader has five options. i.e. five reasonable strategies. He can stock the items from 4 to 12 units. There is no reason to stock more than 12 units as he can never sell more than 12 and there is no possible reason for ordering less than 2. Since there are five alternative courses of action for stocking and five levels of demand, it follows that there are 5 combinations of one strategy and one level of demand. For these 25 combinations. The trader's payoff can be determined in the form of payoff matrix. As per the cost information given, the payoffs are determined for the two situations, i.e. (a) for the demand not more than the stock level and (b) for the demand more than the stock quantity, i.e..

Payoffs for	Q≥D	Q<D
Cost of item	-50Q	-50Q
Sale of item	100D	100Q
Goodwill cost	-	-30(D-Q)
Salvage value	20(Q-D)	-
Carrying cost	-5(Q-D)-5D/2	-5Q/2
Total payoff	-35Q+82.5D	77.5Q-30D

The payoff matrix will be five by five. Each element of the matrix can be determined by above total payoffs for demand less than equal to or greater than the order size. When demand is less than or equal to the order size, following contribution would be obtained to the payoff.

(a) The trader purchases the items for Rs. 50Q.
(b) He sells D of them for Rs. 100D.
(c) He earns salvage of Rs. 20(Q-D) for items not sold.
(d) He acquires the carrying cost of Rs. (0.10)(50)(Q-D) on unsold items, and
(e) He acquires an average carrying cost of (0.10) (50) (D/2) on the items sold during the period.

The total payoff, thus comes out to be as

(a) -35Q+82.5D for demand less or equal to order size.
(b) 77.5Q-30D for the demand more than the order size.

The payoff matrix is given as in table 2:

Unit Demanded (D)						
		2	**3**	**4**	**5**	**6**
Units stocked or order size (Q)	4	Rs. 190	130	70	10	-50
	6	120	285	225	165	105
	8	50	215	380	320	260
	10	-20	145	330	475	415
	12	-90	75	240	405	570
Probability of demand		0.30	0.20	0.30	0.15	0.05

Table-2

The expected payoff can now be determined for the order size. The procedure for calculating the expected values is as following:

For any given strategy, each possible payoff for that strategy is multiplied by the corresponding probability of the given level of demand and all these products are added up. Thus, for first strategy of order size 4 units, the expected value of payoff is:

(190)(0.30)+(130)(0.20)+(70)(0.30)+(10)(0.15)+(-50)(0.05)=Rs. 144

Proceeding similarly, all the expected values can be calculated as following:

Order Size Q	4	6	8	10	12
Expected value (Rs.)	144	190.5	233	214	149.15

Table-3

The objective is to select the strategy that provides the highest payoff. Hence the trader should order for 8 units for the highest expected payoff of Rs. 233.

Out of the above two methods, incremental analysis provides only the optimum level of purchase quantity and does not indicate about the level of expected profit, whereas payoff matrix method
Provides both the answer, i.e., optimum purchase quantity as well as the optimum expected return. Further, it is easy to directly convert the payoff matrix to the opportunity cost matrix as following:

a. When the payoffs are profits, any column of the payoff matrix corresponding to a specific level of demand is taken and the largest payoff to get the corresponding opportunity costs.
b. When payoffs are costs the smallest payoff is taken and smallest payoff is subtracted from each payoff in the same column to get the opportunity costs

In the above example, the opportunity cost matrix can be formed as in the given table 4:

Unit Demanded (D)						
		4	**6**	**8**	**10**	**12**
Units stocked or order size (Q)	4	Rs. 0	155	310	465	620
	6	70	0	155	310	465
	8	140	70	0	155	310
	10	210	140	70	0	155
	12	280	210	140	70	0
Probability of demand		0.30	0.20	0.30	0.15	0.05

Table-4: Payoff matrix for opportunity costs

c. The expected opportunity costs are determined for each alternative with the objective to select the strategy giving minimum expected opportunity cost. The expected opportunity cost for the first alternative is
(0)(0.30)+(155)(0.20)+(310)(0.30)+(465)(0.15)+(620)(0.50)=Rs. 194.75

Similarly, all the expected values can be calculated as given below

Order Size Q	4	6	8	10	12
Expected value (Rs.)	194.75	137.25	94.75	119.75	178.5

Table-5

The decision would be to select the minimum expected costs, i.e. the trader should store 8 units for the lowest costs of Rs. 94.75.

A relationship between the expected opportunity costs and expected payoffs can be determined as follows:

Expected opportunity costs (EOC) = K (constant)-expected payoff (EP)

K=sum of the expected value of the largest entries in each column of the payoff matrix

=190*0.30+285*0.20+380*0.30+475*0.15+570*0.50=327.75 ………. (8)

Thus EOC= K(327.75)-Expected payoff for each strategy

From the equation (8) , it can be observed that the minimum value of EP will simultaneously produce the minimum of EOC. Thus the two analyses, i.e., the payoff matrix method and the opportunity cost matrix method give the same result. If the original payoff matrix is in terms of costs, the relationship (8) will be of the form:

EOC=EP-K, where EP is in the terms of costs.

CONCLUSION

In this paper, a new technique is introduced to solve inventory problem. This method is applicable to solve probabilistic inventory problems. In this method we do not require so much calculation, so this technique is systematic, easy to apply and consume less time in comparison to another techniques.

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