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**ORIGINAL ARTICLE**

# Thermoblackhole Mechanics

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**ABSTRACT**
*Introducing one with the laws of blackhole mechanics, we briefly review the remarkable analogy between ordinary thermodynamics and blackhole mechanics. We also discuss the necessity and validity of the generalized second law(GSL). We explain the flaws arises when one attempt to draw an analogy between the laws of thermodynamics and the laws of blackhole mechanics.*
***Key words:*** *Blackhole , Event horizon, Thermodynamics, Surface gravity, GSL.*

## INTRODUCTION

One of the most remarkable developments in theoretical physics that has occurred in the past forty years, was undoubtedly the discovery of the close relationship between the certain laws of the ordinary thermodynamics and the laws of blackhole mechanics. The starting point of these remarkable developments was the discovery of the four laws of blackhole mechanics by Bardeen, Carter and Hawking [1]. It appears that the laws of blackhole mechanics and the laws of thermodynamics are two major pieces of a puzzle that fit together so perfectly that there can be little doubt that this 'fit' is of deep significance. The existence of this close relationship between these laws seem to be guiding us towards a deeper understanding of the fundamental nature of spacetime, as well as understanding of some aspects of the nature of thermodynamics itself [2].
It was first pointed out by Bekenstein [3] that a close relationship might exist between the certain laws satisfied by blackholes in classical general relativity and the ordinary laws of thermodynamics. He noted that the area theorem of classical general relativity is closely analogous to the statement of the ordinary second law of thermodynamics. His proposal was confirmed by Bardeen, Carter and Hawking [1] , they proved that in general relativity, the surface gravity , $\kappa$ , of a stationary blackhole must be constant over the event horizon,which is analogous to the zeroth laws of thermodynamics. The analogue of the first law of thermodynamics was also proved.
In this paper we have to an attempt to parallel discuss between the laws of blackhole mechanics and thermodynamics. In section-2 we explain the nature of gravity and thermodynamics, two different but there may have an unseen significant relations between them. In section-3 we give precisely the laws of blackhole mechanics. In thermodynamic system containing a blackhole, the generalized second law prevents from the failures of the second laws of thermodynamics and it also give the consistency of blackhole's radiation when quantum mechanics taken into account. We discuss this facts and the validity and problems with this law in section-4. In section -5 we have been drawn an analogy between the laws of blackhole mechanics and thermodynamics. We resolved the problems arises from this analogy by Bekenstein's proposal.
**2. Thermodynamics and Gravity:** Thermodynamics is a branch of physics which deals with the energy, heat, work and entropy of a system. It was born in 19th century as scientists were first discovering how to build and operate steam engines. Thermodynamics deals only with the large scale responds of a system which we can observe and measure in experiments. It is closely related to statistical mechanics from which many thermodynamic relationships can be derived. While dealing with process in which systems exchange matter or energy ,classical thermodynamics is not concerned with the rate at which such processes take place, termed kinetics. For this reason ,the use of the term 'thermodynamics' usually refers to equilibrium thermodynamics. In this connection a

central concept in thermodynamics is that of ' quasi-static processes' which are idealized' infinitely slow' processes. Time dependent thermodynamic processes are studied by non-equilibrium thermodynamics.

The ordinary laws of thermodynamics are of very general validity and they do not depend upon the details of the underlying 'microscopic dynamics' of particular systems. This mean that they can be applied to systems about which one knows nothing other than the balance of energy and matter transfer between them and the environment. Example of this include Einstein's prediction of spontaneous emission around the term of the $20^{th}$ century and the current research into the thermodynamics of black holes. On the other hand, gravitation or gravity is a natural phenomenon in which objects with mass attract one another. Gravitation is most familiar as the agent that gives weight to objects with mass and causes them to fall to the ground when dropped. It is one of the four fundamental force of nature, along with the nuclear force or strong force, electromagnetic force and weak force. Einstein describes gravitations using the general theory of relativity, in which gravitation is a spacetime curvature instead of a force. He proposed that spacetime is curved by matter, and that free falling objects are moving along locally straight paths in curved spacetime.

From the above discussion it is clear that the topics of thermodynamics and gravity lead a rather separate existence in physics. In the broadest sense, thermodynamics regulates the organization of activity in the universe, and gravity controls the dynamics, at least on the large scale. The interaction between these conceptually dissimilar aspects of fundamental physics is still now full of paradoxes, muddle and uncharted hazards. The main difficulties about the thermodynamics of gravitating systems is the apparent absence of true equilibrium. Stars are hot, self-gravitating balls of gas inside which the weight of the star is supported by its won internal kinetic or zero-point quantum pressure. A star is made hotter, not by adding energy, but by removing it, which is unlike ordinary thermodynamic systems.

**3. Blackhole and thermodynamics :** Over the last forty years, blackholes have been shown to have a number of surprising properties. This properties have revealed unforeseen relations between the otherwise distinct areas of general relativity, quantum mechanics and statistical mechanics. This interplay, in turn, led to a number of deep puzzles at the very foundations of physics. Some have been resolved while others continue still now. The thermal properties of blackholes come from the behavior of their macroscopic properties that were formalized in the four laws of black hole mechanics by Bardeen, Carter and Hawking [1]. They dictate the behavior of blackholes in equilibrium , under small perturbations away from equilibrium , and in fully dynamical situations. Although, these laws are consequences of classical general relativity alone, but they have a close similarity with the laws of ordinary thermodynamics.The origin of this seemingly strange coincidence lies in quantum physics. Although this parallel was extremely suggestive, taking it seriously would require one to assign a non-zero temperature to a blackhole, while all agreed was absurd because blackhole by its very definition do not emit anything, so the only temperature one might be able to assign them is absolute zero. But this idea was overthrown by the discovery of Hawking radiation. He proposed that blackholes are not completely black and their physical temperature are not absolute zero [4] .The surface gravity of blackholes can indeed be interpreted as a physical temperature.

At first in 1971, Hawking stated that the area , A of the event horizon of a blackhole can never decrease(but can remain constant) in any process;

$\Delta A \geq 0$ ..........................................(1)

When radiation or matter falls through it, or when two blackholes coalesce, there is an increase in the total horizon area. In this respect it is much like the thermodynamic concept, entropy. The entropy of the universe can increase, but it can never decrease. It was later noted by Bekenstein[3] that this result is analogous to the statement of the ordinary second law of thermodynamics, namely that the total entropy , S of a closed system never decrease in any process;

$\Delta S \geq 0$ .......................................(2)

The above comparison suggests that it might be useful to consider blackhole physics from a thermodynamic view point; something like entropy may also play a role in it. The difference of these two laws are ; in thermodynamics one can transfer entropy from one system to another and it is required only that the total entropy does not decrease whereas in the case of blackhole, one cannot transfer area from one blackhole to another since blackholes cannot bifurcate. So the second law of black hole mechanics requires that the area of each individual blackhole does not decrease in any process. In this sense the second law of blackhole mechanics is slightly stronger than the corresponding thermodynamic law.

Bekenstein realized that considerable information was lost within the event horizon when the blackhole was formed. He suggested that the entropy of the blackhole could be related to the logarithm of this information. This information is , in fact , related to the surface area and it was eventually shown that the entropy of a black hole $S_{bh}$ could be written

as ; $S_{bh} = \frac{c}{4\hbar} A k_B$ ..........................................(3) , where A is the surface area of the event horizon, $\hbar$ is the Planck-Dirac constant $(\frac{h}{2\pi})$ , $k_B$ is Boltzman's constants.

Hawking discovered that the surface of a blackhole could not have a temperature of absolute zero. Mathematically it appeared to have a non-zero temperature. Hawking discovered by applying quantum mechanics to the region near the event horizon, that blackholes can emit all species of particles and radiation [5].

In particular , the spectrum of emission is given by [6],

$$< n > = \frac{\Gamma}{e^{\frac{\hbar\omega}{kT}} - 1} \qquad \text{..................................(5)}$$

Where $< n >$ is the mean number of quanta emitted in one mode of frequency $\omega$ , and $\Gamma$ is the blackholes absorbivity. The surface temperature of black hole is given by;

$T = \frac{\hbar\kappa}{2\pi\, c k_B}$ ..........................(6), where $\kappa$ is the surface gravity of the blackhole evaluated on the event horizon [5] . After established that the blackholes have a non-zero surface temperature and an entropy it is easy to show that they also obey the zeroth, first and second laws of thermodynamics. It is also believed that they may also obey the third law in most but not necessarily all cases. To obey it in all cases requires that the 'cosmic cencorship hypothesis' be satisfied.

**(a) zeroth law of black hole mechanics:** This law states that " The surface gravity , $\kappa$ of a stationary black hole is constant over the event horizon". Although $\kappa$ is defined locally on the event horizon, it turns out that it is always constant over the horizon of a stationary blackhole. This constancy is reminiscent of the zeroth law of thermodynamics which states that the temperature is constant throughout a body in thermal equilibrium. It suggests that the surface gravity is analogus to the temperature. T constant for thermal equilibrium for a normal system is analogous to $\kappa$ constant over the event horizon of a stationary blackhole. The surface gravity is related to the physical temperature of the blackhole , namely Hawking temperature is given by [5] ,

$$T_H = \frac{\hbar\kappa}{2\pi k_B} \qquad \text{..............................(7)}$$

For the case of Schwarzschild black hole, where $\kappa = \frac{1}{4GM}$ ,the Hawking temperature becomes;

$$T_H = \frac{\hbar}{8\pi G k_B M} \approx 6.2\times 10^{-8} (\frac{M_{\otimes}}{M})^0 K \qquad \text{.............................(8)}$$

So this is completely negligible for solar mass black hole- the black hole absorbs much more from the microwave background radiation than it radiates itself.

**(b) First law of black hole mechanics:** This law deals with the mass change, $dM$ when a black hole undergo from one stationary state to another. Mathematical formulation of this law is given by;

$$dM = \frac{\kappa}{8\pi} dA + work\ \ term \qquad \text{.....................................(9) or,}$$

$$dM = T_H dS_{bh} + work\ term \qquad \text{....................................(10)}$$

The 'work terms' depends on the type of blackholes. For the most general type Kerr-Newman black hole family, the first law takes the form;

$$dM = \frac{\kappa}{8\pi} dA + \Omega\, dJ + \Phi\, dQ$$ ......................................(11) , where $\Omega$ is the angular velocity and $\Phi$ is the electric potential which are given by

$$\Omega = \frac{\delta M}{\delta J}$$ .............................................(12) and

$$\Phi = \frac{\delta M}{\delta Q}$$ .............................................(13)

**(c) Second law of black hole mechanics:** The second law of black hole mechanics is Hawking area theorem[7].This law states that, in any classical process the area of the event horizon does not decrease with time i. e.

$$dA \geq 0$$

$or$ ..........................................(14)

$$dS_{bh} \geq 0$$

This law implies for instance that the area of a blackhole resulting from the coalescence of two parent blackholes is greater than the sum of areas of the two parent blackholes. It also implies that the blackholes cannot bifurcate, namely a single blackhole can never split in two parts.

**(d)Third law of black hole mechanics:** The third law of blackhole mechanics states that,'it is impossible by any procedure, no matter how idealized, to reduce $\kappa$ to zero by a finite sequence of operations.' This law has a rather different status from the others, in that it does not, so far at least, have a rigorous mathematical proof [1]. However ,for example if one tries to reduce $\kappa$ of Kerr black hole by throwing in particles to increase the angular momentum, one finds that the decrease of $\kappa$ per particle thrown in gets smaller and smaller as the mass and angular momentum tend to the critical ratio $\frac{J}{M^2} = 1$ i.e. extremal case for which is $\kappa$ zero. Actually $\kappa = 0$ is merely an idealized case because it is forbidden by the 'cosmic censorship hypothesis'.

**4. Generalized second law (GSL):** The correspondence between the laws of ordinary thermodynamics and the laws of blackhole mechanics was treated as a mathematical curiosity without any physical implications, in a seminal paper by Bardeen, Carter and Hawking[1]. At around the same time , Bekenstein [3] was advocating a rather more radical approach. On the basis of blackhole's area theorem he proposed that , multiplied by appropriate powers of the Planck length, Boltzmann constant and some dimensionless constant of order unity, the blackhole area should be interpreted as its physical entropy. This proposal was given physical support by the discovery of Hawking [4]. hat the blackholes radiate at a temperature $T_H = \frac{\hbar\kappa}{2\pi}$ .

Wheeler provided the initial motivation for Bekenstein's blackhole entropy proposal [8] . Wheeler suggested a creature, subsequently called Wheeler's demon, which could violate the ordinary second law of thermodynamics by dropping entropy into a blackhole, producing a decrease in the entropy outside the blackhole. This led Bekenstein to conjecture that the blackhole itself has an entropy.

Wald [2] gives an explanation which further strengthed the physical connection between the laws of blackhole mechanics and the laws of thermodynamics by the following considerations. If we take into account the 'back reaction' of the quantum field on the blackhole, then it is clear that if energy is conserved in the full theory, an isolated blackhole must lose mass in order to compensate for the energy radiated to infinity in the particle creation process. As a blackhole thereby "evaporates", the blackhole entropy $S_{bh}$ will decrease, in violation of the second law of blackhole mechanics. On the other hand , there is a serious difficulty with the ordinary second law of thermodynamics when blackholes are presents: one can simply take some ordinary matter and drop it into a blackhole, where, classically at least, it will disappear into a spacetime singularity. In this later process, one loses the entropy initially present in the matter, but no compensating gain of ordinary entropy occurs, so the total entropy , S , decreases. It is notable that in the blackhole evaporation process, although $S_{bh}$ decreases, there is significant amount of ordinary entropy generated outside the blackhole due to particle creation. Similarly , when ordinary matter is dropped into a blackhole, although S decreases, by the first law of blackhole mechanics , there will necessarily be an increase in $S_{bh}$.

The above considerations motivated Bekenstein to take the following proposal [3] [9]. Although the second law of blackhole mechanics breaksdown when quantum process are considered, and the ordinary second law of thermodynamics breaksdown when blackholes are present, perhaps the following law, known as the generalized second law (GSL) always holds. The law stated that, " In any process, the total generalized entropy never decreases" . This statement means that we must regard blackhole entropy as a genuine contribution to the entropy content of the universe [3]. If we define the total generalized entropy by $S'$ then

$S' = S_{bh} + S_c$ ................................(15)

where $S_{bh}$ is the blackhole entropy and $S_c$ is the common entropy in the blackhole exterior and then GSL becomes

$\Delta S' = \Delta(S_{bh} + S_c) \geq 0$ ....................(16)

Although $S_{bh}$ and $S_c$ individually may decrease, it appears to be true that $S'$ never decreases. If we decrease $S_c$ by throwing matter into a blackhole, we correspondingly increase A i.e. $S_{bh}$ so that $S'$ does not decrease. On the other hand, if A i.e. $S_{bh}$ decreases due to the quantum particle creation processes then the thermal spectrum of the created particles increase $S_c$; again $S'$ does not decrease. Thus neither the second law of thermodynamics nor the blackhole area theorem are satisfied individually, but it appears that we have a new law of physics namely GSL.

The generalized entropy (15) and the generalized second law (16) have obvious interpretaions: Presumably, for a system containing a blackhole, $S'$ is nothing more than the " true total entropy" of the complete system , and (16) is then nothing more than the "ordinary second law" for this system. If so, then $S_{bh}$ truly is the physical entropy of a blackhole.

**4.1 Validity of GSL:** The GSL plays a fundamental role in blackhole physics. Though a number of analysis [10], [11] [12] [13] have given strong support to the GSL but a simple explicit general proof of this law has not been given until now. Although these analysis have been carried out in the context of general relativity, the arguments for the validity of the GSL should be applicable to a general theory of gravity, provided, of course, that the second law of blackhole mechanics holds in classical theory.

The validity of the GSL for the massless radiation evaporated by an uncharged, non-rotating semi-classical blackhole was almost proved by Zurek [13]. Unruh and Wald [10] stressed the importance of the vaccum polarization and acceleration radiation effects for the validity of the GSL . More general arguments for the validity of this law for slowly evolving blackholes were given by Zurek and Thorne [11] . Also a simple explicit proof of the GSL for quasi-stationary changes of a generic charged rotating blackhole emitting, absorbing, and scattering any sort of radiation in the Hawking semi-classical formalism were given by Frolov and Page[12]. They assumed that the incoming state is a product srate of radiation originating from infinity(i.e. IN modes) and radiation that would appear to emanate from the whitehole region of the analytically continued spacetime(i.e. UP modes), and it is argued that the generalized entropy must increase under unitary evolution. This is an explicit mathematical demonstration of what Zurek, Thorne and Price[14] argued verbally, that the GSL is a special case of the ordinary second law, with the blackhole as a hot, rotating, charged body that emits thermal radiation uncorrelated with what is incident upon it. Sorkin[15] argued on quite general grounds that the (generalized) entropy of the state of the region exterior to the blackhole must increase under the assumption that it undergoes autonomous evolution.

Most of the proofs of the GSL based upon two key assumptions[16]. One of the assumption is that the blackholes might be quasistationary, changing only slowly during its interaction with an environment. It has been conjectured [14] that the GSL also holds , using the Bekenstein-Hawking entropy formula $\frac{A}{4}$ for the blackhole, even for rapid changes in the blackhole, but this has not been rigorusly proved. Another assumption is that the semiclassical approximation holds, so that the blackhole described by a classical metric which responds only to some average or expectation value of the quantum stress-energy tensor. This allows the blackhole entropy to be represented by $\frac{A}{4}$ of its classical horizon. This approximation also implies that the radiation from the blackhole is esentially thermal, with negligible correlations between what is emitted early and late in the radiation, so that one may use the von Newmann entropy $S_{rad} = -tr(\rho \ln \rho)$ ( where $\rho$ is the density matrix) for the entropy of the radiation and yet have it plus $\frac{A}{4}$ for the blackhole to continue to increase[16].

It is notable that if one could violate the GSL for an infinitesimal quasi-static process in a regime where the blackhole can be treated semi-classically, then it also be possible to violate the ordinary second law for a corresponding process
involving a self-gravitating body. For example, suppose that the GSL could be violated for an infinitesimal quasi-static process involving, say, a Schwarzschild blackhole of mass M(with M much larger than the Planck mass). This process might involve lowering matter towards the blackhole and possibly dropping the matter into it. However , an observer doing this lowering or dropping can examine only the region outside of the blackhole, so there will be some $r_0 > 2M$ such that the detailed structure of the blackhole will directly enter the analysis of the process only for $r > r_0$. Now replace the blackhole by a shell of matter of mass M and radius $r_0$ and surround this shell with a "real" atmosphere of radiation in thermal equilibrium at the Hawking temperature as measured by an observer at infinity. Then the ordinary second law should be violated when one performs the same process to the shell surrounded by the "real" thermal atmosphere as one performs to violate the GSL when the blackhole is present. Indeed , the arguments of [11] [14] [17] do not distinguish between infinitesimal quasi-static process involving a blackhole as compared with a shell surrounded by a thermal atmosphere at the Hawking temperature. Wald [18] conclude that there appear to be strong ground for believing in the validity of the GSL.

**5. Analogy between black hole mechanics and thermodynamics:** Mathematically, the laws of blackhole mechanics completely analogous to the laws of ordinary thermodynamics. Although the nature of the laws of blackhole mechanics is completely different from the nature of the laws of thermodynamics, so it is generally believed that the analogy between them is purely a mathematical curiosity. But the discovery of particle creation by blackholes and their evaporation suggest that there may be a deep connection between blackhole mechanics and thermodynamics.

The analogy with thermodynamic behavior is striking, with the horizon area playing the role of entropy. This analogy was vigorously persued as soon as it was recognized at the beginning of the 1970's.However, the caution should be used in developing the analogy, it appeared at first some flaws such as;

(i) the temperature of a blackhole vanishes.
(ii) the entropy is dimensionless, whereas horizon area is a length squared.
(iii) the area of every black hole is separately non-decreasing, whereas only the total entropy is non-decreasing in thermodynamics.
(iv) the GSL can be violated by adding entropy to a blackhole without changing its area.

At the purely classical level, it thus appear that the GSL is simply not true. However , when $\hbar \to 0$, the Bekenstein entropy $\frac{\eta A}{\hbar G}$ diverges , and an infinitesimal area change can make a finite change in the Bekenstein entropy. The other flaws [ (i),(ii),(iii)] in the thermodynamic analogy are also in a sense resolved in the limit $\hbar \to 0$. The second flaw is resolved by the Bekenstein's postulate(multiplied by appropriate powers of the Planck length, Boltzmann constant and some dimensionless constant of order unity, the blackhole area should be interpreted as its physical entropy) , while third flaw is resolved because a finite decrease in area would imply an infinite decrease in entropy. Furthermore , the first law of blackhole mechanics implies that the blackhole has a Bekenstein temperature $T_B = \frac{\hbar\kappa}{8\pi\eta}$ , which vanishes in the classical limit when $\hbar \to 0$, thus resolving first flaw. The Bekenstein proposal therfore "explains" the apparent flaws in the thermodynamic analogy, and it suggests very strongly that the analogy is much more than an analogy. It turns out that , with quantum effects included, the GSL is indeed true after all,with the coefficient $\eta$ equal to $\frac{1}{4}$ [19]

The most obvious analogy between blackhole mechanics and thermodynamics is the second law. This law states that the area, A of the event horizon around a blackhole never decrease with time. When two blackholes coalesce, the area of the event horizon around the final blackhole is greater than the sum of the areas of the horizons of the original blackholes,i.e. $A_3 > A_1 + A_2$ .This law shows that the area of the event horizons has a strong similarity to entropy because it is additive and non-decreasing. It is mentioned above that the only difference between horizon area and entropy is that, one can transfer entropy from one system to another but in the case of blackhole one cannot transfer area from one blackhole to another because blackholes can never divide into two, they only joined together .

Consider, the most general case of blackholes i.e. Kerr-Newman blackholes that characterized by mass M, angular momentum J and electric charge Q, the size of the blackhole area A is given by,

$$A = 4\pi(r_+^2 + a^2) = 4\pi[2M^2 - Q^2 + 2M^2(1 - \frac{Q^2}{M^2} - \frac{J^2}{M^4})^{\frac{1}{2}}] \text{.................(17)}$$

With $Q^2 < M^2$ , $J^2 < M^4$ , $G = c = 1$. We see from equation (17) that, it is not clear at a glance whether a disturbance to the blackhole which changes both Q and J as well as mass M, will always increase the total area A. Consider the Penrose energy extraction process from a rotating and charged blackhole by reducing both Q and J. The mechanism of this process is of propelling a small body into the region just out side the event horizon where some particle trajectories possess negative energy relative to infinity .When the body reaches the ergosphere arrange for it to break apart into two fragments in such a way that one of which has negative energy and this part disappears down the hole. As a result it will reduce the total mass M of the blackhole somewhat and the mass-energy thereby released by this sacrificed components appears in the remaining fragment which is ejected to infinity at high speed. During this energy transfer the blackhole's rotation rate is diminished somewhat, so J also decreases. The equation (17) shows that when J decreases, the area A increases but when M decreases, the area A decreases. The changes in M and J are therefore in competition, but a careful calculation shows that J always wins and the area increases[20]. So there is a strong analogy between event horizon area and entropy i.e. the second law of blackhole mechanics and thermodynamics

Now from equation (17) one can obtain;

$dM = \frac{\kappa}{8\pi} dA + \Omega\, dJ + \Phi\, dQ$ ..................................(18), where $\frac{\kappa}{8\pi} \equiv \frac{\delta M}{\delta A}$ etc. The equation (18) is just an expression of mass-energy conservation and corresponding to the first law of thermodynamics. The mathematical form of the first law of thermodynamics is,

$dU = TdS - PdV$ ............................................(19) , where U is the internal energy, T is the temperature, P is the pressure, S is the entropy and V is the volume. In equation (19) the term PdV represents the work term whereas in equation (18) the term $\Omega dJ$ represents the work done on the spin and the term $\Phi dQ$ represents the work done on the electric field. So we can re-write the first law of thermodynamics as,

$dU = TdS$ +'work term'.......................................(20)

And the first law of black hole mechanics as,

$dM = \frac{\kappa}{8\pi} dA$ + 'work term'.......................................(21)

Comparing (20) and (21) we see that if A plays the role of entropy S then $\kappa$ plays the role of temperature T, i.e. $\kappa\, dA \sim T\, dS$ ....................................(22)

Also it can be shown that the $\kappa$ is constant across the event horizon surface. So we have an expression of zeroth law analogous to the zeroth law of thermodynamics.

Finally, there is the third law. For the extreme case we have,

$$a^2 + Q^2 = M^2 \; i.e. \; \frac{J^2}{M^4} + \frac{Q^2}{M^2} = 1 \qquad (23)$$

Then $\kappa$ vanishes, although A does not vanished. This corresponds to absolute zero. It is the limiting case of an object which still possesses an event horizon. But the 'cosmic censorship hypothesis' implies the unattainability of 'absolute zero', $\kappa = 0$ , so it plays the role of the third law.

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