International Archive of Applied Sciences and Technology
Volume 3 [2] June 2012: 21 - 26
ISSN: 0976-4828

Website: www.soeagra.com/iaast/iaast.htm

Original Article

Determination of Seismic Moment Tensor by Deconvolution Method

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ABSTRACT

This work focuses mainly on the determination of seismic moment tensors using deconvolution method. A triangular geometry of the ground about a source emitting seismic surface wave signals was considered.
The region was divided into twelve cells of different quality facctors, Q and had on its boundary twenty distinct stations being traversed by seismic wave signals recorded as seismograms.
The source function, X(t) and the effects of the earth structure,$e(t)$ and$q(t)$were considered to be of the form

$X(t) = e^{i(wt+\theta)}$; $e(t) = \dfrac{1}{r}$; $q(t) = e^{-wt/2Q}$

Where w = angular frequency of the wave and r = the distance from the source.
The Green functions $g_{ij}(t)$ which represented the seismogram at the ith station due to the moment tensor component mj was obtained from $e(t)$and $q(t)$as

$g(t) =* e(t) * q(t) = \int_{-\infty}^{+\infty} e(t-T)q(T)dT$

Similarly, the main seismogram u(t) on each station was obtained by convolving x(t) and g(t).
The results of the seismograms and the Green functions obtained formed matrices U and G respectively. The moment tensors m which best matched the observed seismograms was found in a least square sense using generalised inverse of G i.e

$m = (G^T G)^{-1} G^T$

The result of the moment tensor revealed that the seismic signal was emitted from a source which had a fault whose slips are described by superposition of six different couples. Also, the source fault was characterized by six pairs of equivalent body forces of same magnitude but different directions arranged in diffferent orientations. The couples all being very small in magnitudes showed that earthquake generated was of a small magnitude.

Keywords: *Seismic moment tensor, deconvolution,seismogram, earthquake, Green function.*

INTRODUCTION

Earthquakes occur when rock fractures beneath the surface of Earth. There are many types of earthquakes, each with characteristic patterns of vibrations, or tremors. The study of these vibrations is called seismology, after the Greek word for "motion." Some are associated with magma rising towards the surface of a volcano. The vast majority of earthquakes, however, are modelled by seismologists as two faces of rock slipping past each other along an approximately flat internal surface, called a fault. Faults can be seen and touched at the surface, if fault slip extends this far, or if erosion exposes older faults. If slip on a fault breaches the surface, it can be observed at fault scarps, along which one can measure the relative motion of the two rock faces.
Earthquake prediction is a main topic in Seismology. Here, the goal is to know the correlation between the seismicity at a certain place at a given time with the seismicity at the same place, but at a following interval of time [1].
Seismologists have several methods to measure the size of earthquakes. Seismic moment is used by earthquakeseismologists to measure the size of an earthquake. The scalar seismic moment M_0 is defined by the equation $M_0 = \quad AD$, where

- is the shear modulus of the rocks involved in the earthquake (in dyne / cm^2)
- A is the area of the rupture along the geologic fault where the earthquake occurred (in cm^2), and
- D is the average displacement on A (in cm).

The seismic moment of an earthquake is typically estimated using whatever information is available to constrain its factors. For modern earthquakes, moment is usually estimated from ground motion recordings of earthquakes known as seismograms. For earthquakes that occurred in times before modern instruments were available, moment may be estimated from geologic estimates of the size of the fault rupture and the displacement [2].

Seismic moment is the basis of the moment magnitude scale introduced by Hiroo Kanamori, which is often used to compare the size of different earthquakes and is especially useful for comparing the sizes of especially large (great) earthquakes [4].

Despite the difficulty in the prediction of earthquakes, some scientists have made successful predictions. These include [6, 7, 8, 9, 10, 11, 12].

Rock fracture in detectable earthquakes can occur on scales ranging from a few meters to 1000 km, with slips from millimeters to tens of meters, so seismic moment can span many orders of magnitude. Therefore the size of an earthquake is more conveniently expressed as a magnitude on a logarithmic scale. The Richter magnitude M can be estimated from the amplitudes of P, S or surface waves, and is the measure of an earthquake most widely quoted by the news media. As devised by Charles Richter, each step on the magnitude scale corresponds to a 101.5 increase in seismic moment and energy release, approximately a factor of 32. Two steps correspond to a factor of 1000 increase. One meter of slip on a circular fault of radius 10 km gives rise (approximately) to a magnitude M=7.0 earthquake. Assuming that rock stiffness stays constant, a magnitude M=9.0 earthquake would scale up to 10 meters of slip on a circular fault of 100 km radius.

There are several mathematical expressions developed by seismologists for determining the magnitude of earthquakes. For instance, the magnitude of surface waves of shallow earthquakes at epicentral distance can be measured using the equation:

$$M_{sf} = log_{10}(A_{sf}/T) + 1.66log_{10}(\Delta^0) + 3.3 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots .1.0$$

This equation was developed by Bath (1966). $A_{sf}$is the maximum amplitude of the horizontal ground motion. T is the period of the surface waves and M_{sf} is the magnitude of surface wave.

Guthenberg (1945) proposed a formula for determining the magnitude of body waves having period 1 – 5 s as follows:

$$M_b = log_{10}\,({}^{A_b}/_{T}) + 0.01\Delta^0 + 5.9 \dots\dots\dots\dots\dots\dots\dots 2.0$$

Where A_b is the ground vibration amplitude.

The modified Mercalli magnitude scale is used to express the damage caused by an earthquake, rather than the size of the earthquake itself. This magnitude varies from place to place, according to proximity to the fault, local geology and the clustering of man- made structures. Mercalli magnitudes, though often somewhat subjective, are useful for policymakers who wish to relate earthquake type and size to actual societal hazards. In many earthquake-prone regions, the most damaging earthquakes have occurred before 1900, before the advent of drum seismometers. To estimate Richter magnitudes for these historical earthquakes, seismologists combine measurements of fault scarps and other ground disturbance with Mercalli magnitudes based on old damage reports.

METHODOLOGY

All the data used in this work were generated by assuming a triangular ground geometry, enclosing a source emmiting a seismic wave signal. On this geometry were twenty stations from where seismograms were obtainable. In addition to this, the geometry was divided into twelve cells each characterised by a factor Q as seen in figure1 below.

Also, X(t) that is, the source function, e(t) and q(t) , the effect of the earth structure, were considered to be of the forms:

$$X(t) = e^{i(wt+\theta)} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots .3.0$$

$$e(t) = \frac{1}{r} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots .4.0$$

$$q(t) = e^{-wt/2Q} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 5.0$$

where w is the angular frequency of the wave, and r is the distance from the source.

The seismograms U(t) were then generated from convolving X(t) with the green function g(t). The mathematical expression for the convolution is as shown below;

$U(t) = X(t) * S(t) = \int x(t - T)q(t)d\tau$ … … … … … … … 6.0

Where $S(t) = (t) * q(t) = \int x(t - T)q(t)d\tau$ … … … … … 7.0

Combining these, we obtain,

$$U(t) = \frac{4Q^2(1 - i2Q)}{rw^2(1 + 4Q^2)}\left(e^{-wt/2Q} e^{iw(t-\tau+Q)}\right)_b^a \quad \ldots\ldots\ldots 8.0$$

The seismogram generated for the station was the sum of the effects of the cellstraversed by the signal before reaching the station. In other words, the boundary between any two cells of the geometry was considered as a substation on which a value of seismogram was obtained. For instance, the seismogram on the station number 1 is given by:

$$U_1(t) = u_1 + u_2 + u_3 + u_4 + u_5 + u_6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 9.0$$

Where u_1 , u_2 , u_3 , u_4 , u_5 and u_6 are seismograms obtained from the boundaries traversed by the signal before reaching the station using expression.
The Green's functions were also obtained from the expression 10.0 below which was developed from equation 7.0

$$g(t) = \left(\frac{-2Q}{rw} e^{-w\tau/2Q}\right)_b^a \quad \ldots\ldots\ldots\ldots 10.0$$

Indeed, $g_{ij}(t)$ which developed into a matrix of order 6 by 20(6 x 20) was

$$G_{ij} = e_i(t) * q_j(t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 11.0$$

Considering the geometry of twelve cells where b and a are the time limits in each cell obtained from;
$t = \frac{r}{v}$ where v is the velocity with which the seismic signal traversed each cell of quality factor Q. The velocity and the quality factor of each cell used is as given below:

Cell	1	2	3	4	5	6	7	8	9	10	11	12
Velocity	3000	3520	4400	3910	3960	3520	4500	3000	3800	3400	3200	5200
Q	30	31	32	33	34	35	30	32	36	30	38	40

RESULTS AND DISCUSSION

By substituting all the neccessary variables in equations 8.0 and 10.0, the following wereobtained for U(seismograms) and G(the Green's function).
These results form a matrix U as follows;

$$U = \begin{pmatrix} 60.00 \\ 72.07 \\ 97.64 \\ 82.28 \\ 151.53 \\ 1.03 \\ 25.10 \\ 38.41 \\ 1120.89 \\ 67.71 \\ 82.55 \\ 65.55 \\ 42.58 \\ 35.90 \\ 38.36 \\ 80.30 \\ 85.50 \\ 5.29 \\ 59.89 \\ 59.08 \end{pmatrix} X\ 10^{-5}$$

The Green function in matrix form G is as follows:

$$G = \begin{pmatrix} 1.46 & 1.46 & 1.49 & 1.50 & 1.36 & 1.49 \\ 1.78 & 1.78 & 1.82 & 1.83 & 1.66 & 1.81 \\ 2.27 & 2.27 & 2.32 & 2.33 & 2.12 & 2.31 \\ 2.75 & 2.75 & 2.80 & 2.82 & 2.56 & 2.79 \\ 2.56 & 2.56 & 2.61 & 2.63 & 2.39 & 2.60 \\ 1.57 & 1.57 & 1.61 & 1.62 & 1.47 & 1.60 \\ 1.14 & 1.14 & 1.16 & 1.17 & 1.06 & 1.16 \\ 1.41 & 1.41 & 1.44 & 1.45 & 1.32 & 1.43 \\ 1.70 & 1.70 & 1.74 & 1.75 & 1.59 & 1.73 \\ 1.95 & 1.95 & 1.99 & 2.00 & 1.82 & 1.98 \\ 2.05 & 2.05 & 2.09 & 2.10 & 1.91 & 2.08 \\ 1.86 & 1.86 & 1.90 & 1.91 & 1.74 & 1.89 \\ 1.52 & 1.52 & 1.55 & 1.56 & 1.41 & 1.54 \\ 1.36 & 1.36 & 1.39 & 1.40 & 1.27 & 1.39 \\ 1.20 & 1.20 & 1.23 & 1.24 & 1.12 & 1.22 \\ 2.15 & 2.15 & 2.20 & 2.21 & 2.01 & 2.19 \\ 2.41 & 2.41 & 2.46 & 2.47 & 2.25 & 2.45 \\ 1.64 & 1.64 & 1.67 & 1.68 & 1.53 & 1.66 \\ 1.89 & 1.89 & 1.93 & 1.94 & 1.77 & 1.93 \end{pmatrix} X\,10^{-5}$$

The seismic moment tensors m_{js} were then obtained by using the expression:

$M = (G^T G)^{-1} G^T U$……………………..12.0

$$G^T = \begin{pmatrix} 1.46 & 1.78 & 2.27 & 2.75 & 2.56 & 1.57 & 1.14 & 1.41 & 1.70 & 1.95 & 2.05 & 1.86 & 1.52 & 1.36 & 1.20 & 2.15 & 2.73 & 2.41 & 1.64 & 1.89 \\ 1.46 & 1.78 & 2.27 & 2.75 & 2.56 & 1.57 & 1.14 & 1.41 & 1.70 & 1.95 & 2.05 & 1.86 & 1.52 & 1.36 & 1.20 & 2.15 & 2.73 & 2.41 & 1.64 & 1.89 \\ 1.49 & 1.82 & 2.32 & 2.80 & 2.61 & 1.61 & 1.16 & 1.44 & 1.74 & 1.99 & 2.09 & 1.90 & 1.55 & 1.39 & 1.23 & 2.20 & 2.78 & 2.46 & 1.67 & 1.93 \\ 1.50 & 1.83 & 2.33 & 2.82 & 2.63 & 1.62 & 1.17 & 1.45 & 1.75 & 2.00 & 2.10 & 1.91 & 1.56 & 1.40 & 1.24 & 2.21 & 2.80 & 2.47 & 1.68 & 1.94 \\ 1.36 & 1.66 & 2.12 & 2.56 & 2.39 & 1.47 & 1.06 & 1.32 & 1.59 & 1.82 & 1.91 & 1.74 & 1.41 & 1.27 & 1.12 & 2.01 & 2.55 & 2.25 & 1.53 & 1.77 \\ 1.49 & 1.89 & 2.31 & 2.79 & 2.60 & 1.60 & 1.16 & 1.43 & 1.73 & 1.98 & 2.08 & 1.89 & 1.54 & 1.39 & 1.22 & 2.19 & 2.73 & 2.45 & 1.66 & 1.93 \end{pmatrix} X\,10^{3}$$

$$G^T G = \begin{pmatrix} 74.40 & 74.40 & 75.93 & 76.41 & 69.47 & 75.20 \\ 74.40 & 74.40 & 75.93 & 76.41 & 69.47 & 75.20 \\ 76.41 & 76.41 & 77.52 & 78.00 & 70.86 & 77.11 \\ 76.41 & 76.41 & 78.00 & 78.08 & 72.32 & 77.55 \\ 9.58 & 69.48 & 70.80 & 72.32 & 64.82 & 71.57 \\ 75.20 & 75.20 & 77.11 & 77.55 & 71.57 & 76.66 \end{pmatrix} X\,10^{10}$$

$$(G^T G)^{-1} = \frac{1}{\left|G^T G\right|} Adj(G^T G) \ldots\ldots\ldots\ldots\ldots\ldots\ldots..13.0$$

By estimation, $\left|G^T G\right| = 20574\ X\ 10^{10}$

$$Adj(G^T G) = \begin{pmatrix} 84.24 & 40.70 & 73.71 & 43.02 & 76.54 & 36.50 \\ 82.20 & 46.78 & 56.55 & 45.54 & 48.94 & 41.62 \\ 93.27 & 34.26 & 88.45 & 39.62 & 94.62 & 28.77 \\ 78.93 & 26.84 & 74.72 & 49.65 & 105.38 & 34.61 \\ 66.40 & 35.15 & 65.79 & 34.88 & 82.56 & 46.72 \\ 101.10 & 44.61 & 74.40 & 37.16 & 56.87 & 33.25 \end{pmatrix} X\,10^{10}$$

$$G^T G^{-1} = 1/2.057X10^{10}\ X \begin{pmatrix} 84.24 & 40.70 & 73.71 & 43.02 & 76.54 & 36.50 \\ 82.20 & 46.78 & 56.55 & 45.54 & 48.94 & 41.62 \\ 93.27 & 34.26 & 88.45 & 39.62 & 94.62 & 28.77 \\ 78.93 & 26.84 & 74.72 & 49.65 & 105.38 & 34.61 \\ 66.40 & 35.15 & 65.79 & 34.88 & 82.56 & 46.72 \\ 101.10 & 44.61 & 74.40 & 37.16 & 56.87 & 33.25 \end{pmatrix} X10^{10}$$

$$G^T U = \begin{pmatrix} 8811.74 \\ 8811.74 \\ 4349.56 \\ 4298.07 \\ 3976.80 \\ 4325.35 \end{pmatrix} X\ 10^{-10}$$

$$(G^T G)^{-1} G^T U = \begin{pmatrix} 665875.26 \\ 376948.86 \\ 986255.69 \\ 839978.30 \\ 537852.01 \\ 744008.90 \end{pmatrix} X\ 0.49\ X 10^{-10}$$

$$= \begin{pmatrix} 326278.88 \\ 184704.94 \\ 483265.29 \\ 411589.37 \\ 263547.48 \\ 364564.36 \end{pmatrix} X\ 10^{-10}$$

Therefore,

$$\begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \end{pmatrix} = \begin{pmatrix} 326278.88 \\ 184704.94 \\ 483265.29 \\ 411589.37 \\ 263547.48 \\ 36564.36 \end{pmatrix} X\ 10^{-10} \quad Nm$$

CONCLUSION

Indeed moment tensors have certain descriptions they give about the nature of seismic sources. They present a source as that which has a fault upon which the slip can be described by superposition of couples.

The results of obtained in this work for moment tensors reveal that the source has a fault whose slips are described by superposition of six identical couples. Also, the source fault was characterized by six pairs of equivalent body forces (each pair consisting of two forces identical in magnitude but different in direction) arranged in different orientations.

The couples all being very small in magnitude (measured in the unit of 10^{-10} Nm) showed that the Earthquake generated was of small magnitude.

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