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Original Article

Unrestricted Definition for Every Concept Algebra- Lattice of Concepts, Conceptual Distance, Applications (Especially to Prime Numbers)

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ABSTRACT

The proposed structure is not a Cartesian Product of sets. It is not just subsets of spaces spanned by attributes. It is something more: knowledge, which can be extracted from sets of objects and from sets of attributes by using only the operations. We use couples in order to define concepts, because, in this way, we express the relativity of the objects. The four operations are easily implemented in the computer. We find the concepts successively and we are not obliged to store in the memory (of the computer, of our mind) every concept, but only those we are, at present, interested in. We can create the others whenever we want, taking advantage of the classes and of the fact that, every concept creates (is) a classification (Application2 on Quality Control).

Our operations, which are original, construct a lattice "richer" than others, since it is consisted of more concepts, more possible situations. All concepts are accepted, not only the standardized. Professor R. Wille takes as concepts only the standardized ones. Obviously, this fits well with robots, but not with human beings. Freedom and fantasy can give birth to many partly different or absolutely new objects-we do not have the right to exclude them. Our proposed system of concepts is open to new concepts.

Especially in Medicine, with diseases changing from the one set of symptoms to the other set, from the one term(name) to the other term, with many new-hidden-intermediate levels of interrelations among them, one cannot work only with definitions of normalized diseases, but with paths of symptoms(concepts)!... (Application1 on Medicine).

Application3 on Prime Numbers has the most enthusiastic results: a. the lattice order is given, through the Mathematical Theory of Concepts, to the set of Natural Numbers, b. the prime numbers are generated as distances("differences") inside the lattice-order structure, c. we cannot find a deterministic mathematical formula or rule in order to compute the prime numbers, d. as the natural numbers are increasing, the possibility a prime number to be found is decreasing.

KEY WORDS*: concept, lattice, prime numbers, computation, numbers' structure, medicine, quality control.*

MATHEMATICAL SRTUCTURE OF CONCEPTS

Definition 1. Concept is every assignment of a prototype to an icon, whatever may be the prototype and the icon. We call the prototype "object" and the icon "attributes". We symbolize a concept with a couple whose left part is the object and right part the attributes.

Definition 2. $(O_1, A_1) \dot{\cup} (O_2, A_2) = (O_1 \cup O_2, A_1 \cap A_2)$, where $\cup$ and $\cap$ are the usual operations between sets, union and intersection, respectively. We call this operation "union of concepts".

Definition 3. $(O_1, A_1) \dot{\cap} (O_2, A_2) = (O_1 \cap O_2, A_1 \cup A_2)$. We call this operation "intersection of concepts".

With the above two operations, for every two concepts, there exist a "higher" and a "lower" concept.

Definition 4. $(O_1, A_1) \dot{\subseteq} (O_2, A_2) \Leftrightarrow (O_1 \subseteq O_2$ and $A_1 \supseteq A_2)$, where $\subseteq$ and $\supseteq$ denote the usual subset and superset, respectively. The subordinated concept (O_1, A_1) are the species and the superordinated concept (O_2, A_2) is the genus.

Definition 5. Two concepts (O_1, A_1) and (O_2, A_2) are equivalent if, and only if, $A_1=A_2$. In symbols, $(O_1, A_1) \approx (O_2,A_2) \Leftrightarrow A_1 = A_2$.

Since $\approx$ is an equivalence relation, we have classes of concepts. As we know, the classes are disjoint sets and their union makes the set of reference. So, in our case, we have a partitioning of the set C of all concepts (neither of the set Ω of isolated objects nor of the potential set P (Ω) of objects).

Definition 6. The complement of the concept (O, A) is the concept (O^C, A^C), where O^C and A^C are the usual set-theoretic complements of O and A, respectively.

Definition 7. The symmetric-difference of two concepts (O_1, A_1) and (O_2, A_2), is the concept D= ($O_1 \dotplus O_2$, $(A_1 \dotplus A_2)^C$), where $O_1 \dotplus O_2$, $A_1 \dotplus A_2$ are the usual set-theoretic symmetric-differences of O_1 and O_2 , A_1 and A_2, respectively.

The set C of all concepts, with the two operations intersection ($\dot{\cap}$) and symmetric-difference D is proved to have the order of a Lattice, which is equivalent to the structure of a Boolean Algebra .

Definition 8. We call distance d(X,Y) of two sets X and Y, the non-negative integer expressing the number of elements of the set $X \dotplus Y$, that is of their symmetric-difference (in symbols $n(X \dotplus Y)$). So, $d(X,Y) = n(X \dotplus Y)$.

The three properties for the mathematical definition of a "distance" hold.

An example with distances: suppose we have two sets: A and B, with n(A)=5, n(B)=7.

If $A \cap B=\emptyset$, then the two sets are foreign to each other(they have not common elements).

So, A-B=A, B-A=B, $A \dotplus B=(A-B) \cup (B-A)=A \cup B$, $n(A \dotplus B)=$**$n(A \cup B)=n(A)+n(B)-n(A \cap B)$**$=5+7-n(\emptyset)=12-0=12$. The sum n(A)+n(B) is the maximum possible value of $n(A \dotplus B)$, not only in our example, but generally. Indeed, $n(A \dotplus B)=n(A-B)+n(B-A)-n((A-B) \cap (B-A))$ [because of the above formula with the bold letters] = $n(A-B)+n(B-A)-n(\emptyset)= n(A-B)+n(B-A)-0= n(A-B)+n(B-A) \leq n(A)+n(B)$, where the equality holds iff (A-B=A and B-A=B) , equivalently, iff $A \cap B=\emptyset$.

If A is a real subset of B, then $n(A \dotplus B)=n(A-B)+n(B-A)-n((A-B) \cap (B-A)) =n(\emptyset)+ n(B-A)-n((A-B) \cap (B-A))=0+n(B-A)-n(\emptyset \cap (B-A))= n(B-A)-n(\emptyset)= n(B-A)-0= n(B-A) \leq n(B)$, where the equality holds iff A= Ø.

In our example, $n(A \dotplus B)= n(B-A)=2$.

If A=B, then $n(A \dotplus B)=n(A-B)+n(B-A)+0=n(\emptyset)+ n(\emptyset)+0=0$.

We introduce the index $I= n(A \dotplus B)/ n(A \cap B)$, that is the ratio of the dissimilarities to the similarities. In the case n(A)=5, n(B)=7 and $n(A \cap B)=3$, we have $n(A \dotplus B)=2+4=6$ and so I=(2+4)/3=6/3=2**>1**. If n(A)=5, n(B)=4 and $n(A \cap B)=$ =3, we have $n(A \dotplus B)=2+1=3$ and so I=3/3**=1** (the dissimilarities are as many as the similarities). If A=B≠Ø, then n(A)≠0 and I=0/n(A)**=0.** If A and B foreign to each other, then, at least one from A and B is≠Ø, $n(A \dotplus B)=n(A)+n(B)>0$, while $n(A \cap B)=$ =n(Ø)=0 and so I**=+∞. We remember, now, the ratios σ/μ from Statistics and HDL/LDL from Medicine!**

Let's go, now, to the concepts. We can take $d(O_1, O_2)=n(O_1 \dotplus O_2)$, which is a distance between objects, but it does not say many things, since it is quantitative but not qualitative: two sets of objects may have many different elements, coming from the same homogenous population (Biometry, Psychometry, students' and teachers' evaluation ....). Besides, we are not working with objects or attributes, but with both of them, that is concepts. The symmetric-difference $O_1 \dotplus O_2$ of the objects, has the icon $(A_1 \dotplus A_2)^c$. So, if we want the real distance of O_1 and O_2, we must check $(A_1 \dotplus A_2)^c$.

$d(A_1,A_2)=n(A_1 \dotplus A_2)=n(\Omega') - n((A_1 \dotplus A_2)^c)$, where Ω' is the set of all attributes (in our certain application). So, $n((A_1 \dotplus A_2)^{\circ})=n(\Omega') - d(A_1,A_2)$. n(Ω') is a constant. Consequently, if the distance of the attributes is increasing, $n((A_1 \dotplus A_2)^c)$ is decreasing and the distance of the objects is, accordingly, decreasing. The explanation comes naturally: if we have a large range of attributes, this range can fit only to a small range of objects (larger intension, smaller extension). It is the same with statistical analysis by zones.

How can we succeed to have small distance between two objects? When $d(A_1, A_2)$ is large, or, equivalently, when $n(A_1 \overset{+}{\bullet} A_2)$ is large, which refers to the intersection $(A_1 \cap A_2)$ and «the area out of $A_1 \cup A_2$», that is the set $(A_1 \cup A_2)^c$. This second set is a **fuzzy factor** in the **definition** or comparison of concepts.

If it is large, then $d(O_1, O_2)$ is small, maybe $d(O_1, O_2) \approx 0$, which means, not that $O_1=O_2$ (*on the contrary!...*), but that O_1 and O_2 *can be easily confused and considered as equal*! In many applications, we do not know exactly O_1 , O_2 , A_1 , A_2, but only their **common** elements and their **differences.** Instead of «fishing» (*stochastically!...*) in the «area» of $(A_1 \cup A_2)^c$, is better to try to maximize $A_1 \cap A_2$ (*except if we take the risk ...*).

The more attributes two objects have in common, the more they are alike, or almost equal, that is $d(O_1, O_2) \approx 0$. In the same time, we try to minimize or, at least, keep below a certain level, the «area» $(A_1 \cup A_2)^c$ (since, we can never extinguish this fuzzy factor). It is exactly what happens in Statistics with the mistakes of I or II kind and the levels of significance.

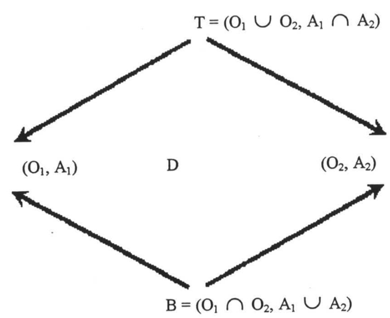

Fig 1. "Children" from two "parents"

where D is the symmetric-difference of (O_1, A_1) and (O_2, A_2). The arrows give the subordinated concepts. (O_1, A_1), (O_2, A_2) and D are located on the same level and, consequently, ***there is no order among them***.

The complement of a concept (O, A) is the concept (O^c, A^c). Attention: (O^c, A^c) has the meaning of the assignment (as all concepts have), but it just says that the common attributes (if any) of the elements of O^c must be found (detected) inside the set A^c. It is not known which of them are exactly. It is, now, obvious that the situation is stochastic, or, even more, fuzzy. Consequently, **enumeration, in this area is impossible**….

Even if we make standardization of the concept (O, A) (which means that O is the maximum set of objects with the minimum set A of common attributes), when we «go out of» O, nothing is sure for the attributes of O^c. The elements of A are common attributes for the elements of O, because (O, A) is constructed from other concepts by using the operation $\dot{\cup}$, which, from its nature, has the meaning of common attributes. If we have a set S, subset of O^c, the only thing we know for sure is that it cannot have A as the minimum set of common attributes. Many people think that, if we make standardizations, then everything will be clear and sure in our world. As a matter of fact, we can be sure only for a finite number of objects, but we cannot be sure for all the others. It depends which objects we select to make standard: then we «loose» all the others!

What does the symmetric - difference say? That the non - common objects $O_1 \overset{+}{\bullet} O_2$ have the common attributes $A_1 \cap A_2$, but they may have, also, others «out of» $A_1 \cup A_2$ (that is, in the complement of $A_1 \cup A_2$). This means that, for foreign sets of objects, we cannot be sure for their common attributes: except of the known ones $A_1 \cap A_2$, there may exist other unknown (because they belong to the complement of $A_1 \cup A_2$).

Conclusions: 1. two concepts can be: a. interrelated either by having common objects or by having common attributes, b. discriminated by discriminating *both* objects and attributes… 2. The symmetric-difference shows us the dissimilarities and the intersection the similarities among our objects.

APPLICATIONS

1. To Medicine

The key idea for the interpretation of the above knowledge structure to Medicine, is to understand diseases as concepts (O,A): the set O of objects consists of term(s) of disease(s) and the set A of attributes consists of its(their) symptom(s)-observed characteristic(s).

According to Figure1., the "children", diseases T and B, come from two "parents"(with the inverse direction of the arrows). So, we have two **new** diseases and we must find(create) their names(terms). E.g., if there exist some dark points("olives") on the skin-which could be simply(?...?) dermatological- and a dry permanent cough- which could be simply(?...?) a result of smoking- then we may speak for cancer in the lungs.

But, if we take the arrows as they are in Figure 1., then we realize that: i. disease B may, not evolve, but recognized-standardized as to be the disease (O_1,A_1) or the disease (O_2,A_2) (from the symptoms $A_1 \cup A_2$, we accept only some of them, A_1 or A_2 , considering the rest as non critical). ii. if we have the "rather few" symptoms $A_1 \cap A_2$ of the disease T, then T may evolve to the disease (O_1,A_1) or to the disease (O_2, A_2).

Besides, we must not forget the "mysterious" third possibility: the symmetric-difference D, that "stands in the middle", expressing the unknown, the unexpected, being the "fear" of the doctors of Medicine!…Three cases: 1."reading" an X-ray picture, or the pictures got by any other image processing technique, is a very difficult task. E.g., how can one discriminate pneumonia from cancer of lungs? To a certain extent, the two pictures are alike. 2. some substances can serve as a poison or a medicine! That's why, in Greece, we say "pharmako" = medicine and "pharmaki" = poison(according to the set of attributes it presents each time). 3. Double-agents(not only in politics but, also, in Biology-Medicine), are a good example of belonging to two different (partly or completely) systems .

Further on, we have more than the three levels of Figure 1., we have **new(hidden) intermediate** levels, we have **classification** of the diseases and **links** between them(fever and cough may "lead to" several different diseases…). *There are not diseased and non-diseased situations, but just paths and nodes in the conceptual graphs..* Very often, we see the birth of partly different or absolutely new objects(diseases, germs and so on!...)-we do not have the right to exclude them. Old and new concepts are connected, there is an order among them, not an hierarchical one but a **nonlinear**, the more complex one of the lattice. Objects and attributes are both necessary for a concept to be formed. A Medical Knowledge Database should include all these concepts, **without trying to separate objects into distinct(however, artificial!...) categories.** Knowledge Databases should have the structure of Lattice, because Lattice is the structure of Knowledge itself...

2. To Quality Control (an application with statistical data)

We have a tobacco company. It produces cigar and cigarettes. Boxes from aluminium or paper are used for both cigars and cigarettes. Boxes for cigarettes are all opening in the usual way (remaining one piece), while boxes for cigars are opening in two ways: one- piece and two- pieces (from above). Cigar boxes opening in two pieces have all the cylinder shape, while all the other boxes have the rectangular shape. The number of cigarettes per box is 20 or 10 for paper or aluminium boxes, respectively. The number of cigars per box is 5 or 10 for rectangular or cylindrical shape, respectively. There are two sizes (normal and kind size) for both cigars and cigarettes.

So, the attributes we test in our samples are the following

Ω = {one-piece opening, two-pieces opening, range of the base, length of the base, width of the base, altitude, paper, aluminium, normal size, king size, number of pieces inside the box, cigar, cigarette}.

Some cases could be the following:

a. A_1 = {one-piece opening, length, width, altitude, paper, normal size, 5 pieces, cigar}

Then, from A_1 we take O_1, which consists of cigar boxes of rectangular shape and paper material, with one- piece opening, containing 5 cigars of normal size.

b. A_2 = {two - pieces opening, range of the base, altitude, aluminium, normal size, 10 pieces, cigar}.
Then, O_2 consists of cigar boxes of cylindrical shape and aluminium material, with two-pieces opening, containing 5 cigars of normal size.
c. A_3 = {one- piece opening, length, width, altitude, paper, king size, 20 pieces, cigarettes}.
Then, O_3 consists of cigarette boxes of rectangular shape and paper material, with one- piece opening, containing 20 cigarettes of kind size. Let's see now, what the following operations mean:

a. $(O_1, A_1) \dot{\cup} (O_2, A_2) = (O_1, \cup O_2, A_1, \cap A_2) = (O_1, \cup O_2$, {altitude, normal size, cigar}).
So, $O_1, \cup O_2$ contains cigar boxes of normal size, for which we measure the altitude.

b. $(O_1, A_1) \dot{\cap} (O_2, A_2) = (O_1 \cap O_2, A_1 \cap A_2) = (O_1 \cap O_2$, {one-piece, length, width, altitude, paper, normal size, 5 pieces, cigar, two-pieces, range of the base, aluminium, 10 pieces}). But one-piece and two-pieces, or 5 pieces and 10 pieces or paper and aluminium or rectangular and cylindrical are contradictory to each other. Consequently, $O1 \cap O_2 = \varnothing$.

c. $(O_1, A_1) \dot{\cup} (O_3, A_3) = (O_1 \cup O_3, A_1 \cap A_3) = (O_1 \cup O_3$, {one-piece, length, width, altitude, paper}).
Then, O, U O_3consists of rectangular paper boxes with one-piece opening.
d. $O_1 \cap O_3 = \varnothing$.
e. For the set $O_2 \cup O_3$, we measure only the altitude.
f. $O_2 \cap O_3 = \varnothing$.

3. To Prime Numbers

How numbers are generated? Is there a deterministic rule by which they are generated, or they appear stochastically (at least, some of them)? We must not always see the natural numbers with their linear order (1,2,...,n,n+1,...), but is rather better to give them a more complex structure: the structure of a lattice. We need three operations: “union” of two numbers, “intersection ” of two numbers and “complement” of a number.

A way to do that is to define as “union” the Least Common Multiple (LCM) and the “intersection” as the Maximum Common Divisor (MCD). But, what is the “complement” of a number? If we have no complement, we have no symmetric-difference, consequently, we have no distance(...), all the natural numbers seem to be ordered by these two operations(but, are, really, all?). Besides, the symmetric-difference plays the role of “addition” in the Algebra Boole which comes from the Lattice Structure(Order). So, something is missing... If we go from the natural numbers to the integers [by using the new characteristic “minus”(-)], then we could say that the complement of x is –x and the distance of x and y is $|x\text{-}y|$, but –x has to do with the linearity of the integers and not with the characteristic of divisibility (like LCM and MCD). In a way, -x expresses quantity but not quality.

At this point, we understand that we must use characteristics and express numbers as concepts[that is, couples (O,A) with their four operations, as defined in the beginning of the present work and in the references 3., 4., 6. and 7]. Every number is an isolated object o and A is its set of attributes [according to the one or more characteristic(s) we use]. By using the four operations, we get other numbers, sets O of numbers and so on (as results of the application of the Mathematical Theory of Concepts). In this way, the set of integer numbers becomes a network (especially, a lattice), not just a line, and unfolds the whole of its richness(in characteristics). Moreover, we can produce classes of numbers, new numbers, classes of classes of numbers, new kinds of numbers and so on. Numbers exist if the appropriate concepts exist. If we have concepts, we get the corresponding numbers. Of course, this happens with every object in our world (“real” or “imaginary”...), not only with the object “number”.

We can define: 1. as set A of attributes of a number o, the set of all its divisors, 2. as A_i UA_j of two sets A_i and A_j of attributes, their usual set-theoretic union, 3. as as complement A^c of a set A of attributes, the set of all numbers, less than o and different from 1, which are not divisors of o (e.g., if o={30}, then A={1,2,3,6,5,10,15,30} and A^c={4,7,8,9,11,12,13,14,16,17,18,19, 20,21, 22,23,24,25,26,27, 28,29}).

Then, it is proved that: 1. $(o_1, A_1) \dot{\cup} (o_2,A_2)$ gives the MCD of the two numbers o_1 and o_2 or, rather, the class of the MCD, 2. $(o_1, A_1) \dot{\cap} (o_2,A_2)$ gives the LCM of the two numbers o_1 and o_2 or, rather, the class of the LCM, **3. the symmetric-difference of (o_1, A_1) and (o_2,A_2) gives the conceptual distance of (o_1, A_1) and (o_2,A_2) and is always a prime number! (conceptual means, from the point of view of**

the characteristic "divisibility" we are examining now and not the Euclidean or any other distance). This is the unique way the prime numbers are generated: not by unions and intersections(which express similarities), but by distances(differences)! ...

Important results:
We cannot find a mathematical formula in order to compute the prime numbers. We can make a computer program, or just an algorithm (like the famous one of the Greek mathematician Eratosthenes), checking sequentially the natural numbers and thus finding sequentially the prime numbers (if runtime is enough...), but **there is no formula returning the prime numbers!** The primes are the non-ordered elements of the system of concepts, consequently we cannot use the other elements to compute the primes..... (the contrary is true: we use just the primes and the usual multiplication between them to construct the others). **The non-ordered cannot be computed and, consequently, cannot be forseen...** They are the naughty children of the system! Of course, we must not forget that, in our lattice structure, multiplication is the intersection and, especially, in the lattice of the natural numbers (as defined above), intersection is the LCM.
For the definition of the symmetric-difference (conceptual distance, prime number), we need the union of two concepts (natural numbers, in our case), the intersection of two concepts and the complement of a concept. But the complement is not a deterministic function, because of the ***fuzzy factor***. In the above example, A^c={4,7,8,9,11,12,13,14,16,17,18,19, 20,21, 22,23,24,25,26,27, 28,29}, in absolute values. Then, where is the fuzziness? This happens, because in the above definition 3. of the complement, we had imposed the restriction " less than o". Without this restriction, we go in "the outer area", where the non-divisors of o exist and their number is infinite!... Consequently, we have infinite number of complements, of symmetric-differences, of prime numbers. **So, we have obtained a stochastic way to find the prime numbers, but not a deterministic formula or rule.**
As the Greek mathematician Euclides has proved, prime numbers are infinite and ever increasing. We understand, now, that: **as they increase, they have less frequency of appearance (the possibility to find a prime is decreasing)**. Really, as the numbers are increasing (in the structure of the lattice), the possibility to find a non-divisor, not already used, becomes smaller.

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