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Original Article

Texture Specific Lepton Mass Matrices and CP Violating Phases

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INTRODUCTION

In 1930, Pauli first postulated the idea of neutrino in order to preserve the conservation of energy, conservation of momentum, and conservation of angular momentum in beta decay

$$n \rightarrow p + e^{+} + \bar{\nu}_e$$

He assumed that neutrino is a neutral weakly interacting particle with spin ½. He originally proposed that the light particle emitted in beta decay is neutron. But by the time he published it in 1933, the neutron had already been discovered by the Chadwick as the neutral partner of the proton. So, it was renamed by Fermi as 'neutrino' in 1934, meaning 'little one'.

In 1942, Kan–Chang Wang first proposed the use of beta capture to detect neutrino experimentally. However, neutrino was detected within Pauli's life time (~887s) by Cowan and Reines[1] in 1956. This was made possible by the advent of nuclear reactors, which provided a rich flux of (anti) neutrinos ($\sim 10^{13}/cm^2/s$). The experiment consisted of two tanks, each containing 200lts of water with dissolved $CdCl_2$ and sandwiched between three scintillation detectors. The proton in the water provided target for the interaction: $\nu_e p \rightarrow e^+ n$. The gamma ray from the annihilation of the produced e^+ and e^- provided the scintillation signal. This was followed closely by gamma ray from neutron absorption in cadmium: $n\,^{48}Cd \rightarrow \gamma\,^{49}Cd$. The observed double γ-ray signal was further confirmed by its correlation with the reactor being in operation. (The Noble prize came to Renies 40 years later in1996).

The successful detection of ν_e was soon followed by another Nobel Prize winning neutrino experiment – i.e. the discovery of second flavor of neutrino ν_μ .It comes from muon decay process as well as from pion decay: $\pi^{\pm} \rightarrow \mu^{\pm}\, \nu_\mu(\bar{\nu}_\mu)$.The high energy neutrinos coming from this decay were bombarded on nucleon target at Brookhaven laboratory experiment, which detected the muon produced via $\nu_\mu n \rightarrow \mu^- p$ ($\bar{\nu}_\mu p \rightarrow \mu^+ n$). This was closely followed by the detection of cosmic ray experiment at Kolar Gold mine. The third neutrino species ν_τ was discovered in 2000 at Fermi lab by observing the τ leptons produced via $\nu_\tau n \rightarrow \tau p$ in nuclear emulsion experiment.

In June 1956, C.N Yang and T.D Lee suggested that weak force might violate parity conservation: $^{60}Co_{27} \rightarrow {}^{60}Ni_{28} + e^- + \bar{\nu}_e$. The result of the Co-60 experiment was formalized in two component theory which says that the neutrino is always left handed (or has left helicity).Later on, it fits nicely with the Gell-Mann and Feynman formulation (1958) of the left handed weak force (also known as V-A theory[2],a form that violates parity maximally). Since the V-A theory says that weak force violate parity maximally as it acts on only left handed states of all quark and lepton whether they have mass or not, hence it led to first theoretical idea about neutrino mass.

In 1933, the first method of the measurement of neutrino mass was proposed by Fermi[3]. He proposed to search for effect of neutrino mass via high energy part of β −spectra. Neutrino mass is searched for through the investigation of beta spectrum of tritium decay: $^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$.The upper bound for ν_e mass found to be $m_{\nu e} \leq 500eV$.With further experiment, this bound was decreasing and at the end of fifties for upper bound of $m_{\nu e}$ found to be $m_{\nu e} \leq (100\text{-}200)eV$.Also ,the upper limit for the mass of τ neutrino can also be inferred from detailed analysis of the shape of high energy end of muon spectrum,$\tau^- \rightarrow \mu^- + \bar{\nu}_\mu + \nu_\tau$.The most accurate value today is $m_{\nu\tau} \leq 70MeV$.

In the standard model neutrinos are assumed to be the exactly massless Weyl particles. This assumption agrees with all direct-mass search experiments, which have set the upper bounds on primary mass eigenstates (ν_1, ν_2, ν_3) of electron, muon, and tau neutrinos [4].

$m_{\nu 1} < 15eV$, $m_{\nu 2} < 0.17$ MeV, $m_{\nu 3} < 18.2$ MeV.

However, the masslessness of neutrinos is not assured by any basic symmetry principle of particle physics. Indeed most extensions of the standard model (such as the grand unified theories) allow the existence of massive neutrinos, although the masses of three active neutrinos may be extremely smaller than those of their corresponding charged leptons.

Recently,a remarkable progress in the studies of neutrino physics has been achieved. The experiments with solar [5-6], atmospheric [7, 8], reactor [9] and accelerator neutrinos [10,11] have provided compelling evidence for the existence of neutrino oscillations. This implies that neutrinos have nonzero masses and that 3-neutrino mixing takes place.

From atmospheric [7, 8], K2K [10] and very recent MINOS data [11], a measurement of the mass squared difference Δm_A^2 has been obtained, in the range $\Delta m_A^2 = 2.5\times10^{-3}eV^2$ at1σ [12]. The atmospheric mixing angle is found to be maximal or nearly maximal $\sin^2\theta_A \geq 0.97(0.87)$ at 1(3) σ. Solar neutrino experiments [5-6] combined with KamLAND results [9] constrain in a narrow range the relevant mass squared difference $\Delta m_S^2 = 7.9\times10^{-5}eV^2$ and the solar mixing angle $\sin^2\theta_s = 0.30$ at1σ[12] . Searches of effects due to the third mixing angle θ_{13} have been so far unsuccessful. The present bound at 3σ read $\sin^2\theta_{13} < 0.041$ [12], from a global analysis of data from the reactor neutrino CHOOZ experiment, the solar and KamLAND experiments as well as of the constraints on the atmospheric mass squared difference from atmospheric and long baseline neutrino data. Many experiments at present and in the future aim to a better precision in the measurement of these parameters.

NEUTRINO MIXINGS AND OSCILLATIONS

A prime motivation for the study of neutrino oscillation has been the long standing “solar neutrino problem”. The theory of dynamics has been in place for long time and has settled down in the form of a “standard solar model”. However, the ^{37}Cl experiment (Davis,1988),which has been running for two decades, has found a consistent discrepancy between the predicted and observed solar neutrino fluxes. During this long period, both the ^{37}Cl experiment and the model of sun have undergone through reinvestigation. There appears to be a real effect in ratio of $\frac{1}{3}$ to $\frac{1}{4}$ between measured versus predicted flux. More recently ,in the water Cherenkov detector Kamiokande II ,an upper bound of about ½ has been established for the same ratio(Mann et al.,1988).Among the many theoretical ideas proposed, neutrino oscillations seem to offer the most reasonable solution.

The possibility of flavor oscillation was first examined by B. Pontecorvo on one hand and by Maki, Nakagawa and Sakata on the other. Neutrino oscillation phenomenon implies that neutrinos produced in a well defined flavor eigenstates can be detected, after propagating a macroscopic distance, as a different flavor eigenstates. The simplest interpretation of this phenomenon is that like all charged fermions, the neutrinos have mass and that similar to quarks, neutrino weak or flavour eigenstates are different from neutrino mass eigenstates i.e. neutrinos mix. For example, an e-type neutrino, produced in the core of the sun propagates as the superposition of three mass eigenstates which pick up different phases as they travel. At the detector these phases are recombine to form flavor states. Because of phase differences introduced during propagation, the recombined wave will have a μ component and a τ component in addition it started with. The phenomenon of neutrino oscillations implies non-zero and non-degenerate neutrino masses as well as the flavour eigenstates being linear combination of mass eigenstates, leading to the Pontecorvo–Maki-Nakagawa-Sakata (PMNS) mixing matrix similar to Cabibbo Kobayashi-Maskawa matrix (CKM) in quark sector.

NEUTRINO OSCILLATIONS IN TWO FLAVORS

The neutrino weak interaction flavor eigenstates are superposition of mass eigenstates which are not observables. Using the usual notations, the three flavor eigenstates ν_a are related to the three mass eigenstates ν_i through the unitary transformation i.e.

$$\nu_a = \sum_i U_{ai}\nu_i ,$$

with $i = 1,2,3$ and $a = e, \mu, \tau$.

The time evolution of a neutrino with momentum $\vec{p}$ produced in a state ν_a at time t=0 is given by,

$$\nu(t) = e^{i\vec{p}.\vec{r}} \sum_i U_{ai} e^{-iE_i t} \nu_{i,}$$

where $E_i = \sqrt{m_i^2 + \vec{p}^2}$ is the energy of neutrinos. Since $m_i << E_i$,

$$E_i \approx p + \frac{m_i^2}{2p} .$$

For the sake of simplicity we first discuss the two flavor case.

In this case, the mixing matrix U is described by one real parameter θ and three phases. The three phases, however can be rotated away by absorbing them in the neutrinos fields. Thus the mixing matrix is given by

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} .$$

Then, we have

$$\begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

This gives

$$\nu_a = \cos\theta\nu_1 + \sin\theta\nu_2 ,$$

$$\nu_b = -\sin\theta\nu_1 + \cos\theta\nu_2 ,$$

The transition probability to detect a neutrino flavor state ν_b at time t can then be calculated to be

$$P_{ab}(t) = \left|\langle \nu_b | \nu(t) \rangle\right|^2 = \sin^2 2\theta \sin^2\left(\frac{(m_2^2 - m_1^2)t}{4p}\right)$$

For survival probability we have

$$P_{aa} = 1 - \frac{1}{2}\sin^2 2\theta\left(1 - \cos\frac{\Delta m^2 t}{2p}\right).$$

In terms of more familiar units the transition probability can be rewritten as $P_{ab} = \sin^2 2\theta \sin^2\left(1.267\frac{\Delta m^2 L}{E}\right) = \frac{1}{2}\sin^2 2\theta\left(1 - \cos 2.53\Delta m^2\frac{L}{E}\right).$

Likewise the survival probability is given by,

$$P_{aa} = 1 - \frac{1}{2}\sin^2 2\theta\left(1 - \cos 2.53\,\Delta m^2\frac{L}{E}\right).$$

In the above equations L=ct is the distance from the source in meters , $E \approx p$ is the neutrino energy in MeV and Δm^2 is neutrino mass squared difference in eV^2.The same relation holds if L is measured in km and E in GeV.

The expression for oscillation length can also be written in the form L

$$L_0 = 2.47\frac{E(MeV)}{\Delta m^2(eV^2)}m.$$

Neutrino oscillation cannot be observed if the oscillation length is much larger than the distance L between neutrino source and neutrino detector. In order to observe the neutrino oscillations, oscillation length must be smaller or of the order of magnitude of length L.

FLAVOUR MIXING MATRIX AND CP VIOLATION IN LEPTONIC SECTOR

Number of fermions mixing parameters

The quark mass matrices in the lagrangian of Yukawa interactions M_u and M_d can be diagonalised by the bi-unitary transformations.

$$M_u = U_{uL}^{+} M_u U_{uR} = Diag(m_u, m_c, m_t)$$

$$M_d = U_{dL}^{+} M_d U_{dR} = Diag(m_d, m_s, m_b)$$

The 3×3 coupling matrix (so called CKM matrix) is given as $U = U_{uL}^{+} U_{dR}$, describes the mixing of quark flavors. For n families of quarks, the n×n flavor mixing matrix U consist of n(n-1)/2 mixing angles and the remaining phase angles. Since the phases of quark field are arbitrary, one can be absorbed. Therefore U can be described in terms of only n^2-(2n-1)=$(n-1)^2$ parameters, among which n(n-1)/2 are the rotation angles and (n-1)(n-2)/2 are the phase angles. For the case n=3, we then arrive at three mixing angles and one nontrivial phase, which is responsible for CP violation.

The charged lepton and Dirac neutrino mass matrices in the flavor basis, M_l and M^D_ν can be diagonalised in a similar way by the bi-unitary transformations:

$$M_l = U_{lL}^{+} M_l U_{lR} = Diag(m_e, m_\mu, m_\tau)$$

$$M_\nu^D = U_{\nu L}^{+} M_\nu^D U_{\nu R} = Diag(m_1, m_2, m_3)$$

In the basis of mass eigenstates, one arrives at 3×3 flavor mixing matrix U in the Lagrangian of charged weak interactions, in which only left-handed leptons take part given as $U^D = U_{lL}^{+} U_{\nu R}$

where U^D is lepton mixing matrix. Similar to the quark mixing case, the 3×3 lepton mixing matrix U^D also have n(n-1)/2 are the rotation angles and (n-1)(n-2)/2 phase angles.

If neutrinos are Majorana particles, however, the situation is different. In this case ,the neutrino mass matrix M_ν^M has the property $(M_\nu^M)^{+}$=M_ν^M i.e. ,M_ν^M is in general a complex symmetric n×n matrix. The diagonalization of M_ν^M needs only a single unitary matrix. Unlike the quark or Dirac neutrino mixing case, there is no freedom to redefine phases of Majorana neutrino fields, as Majorana particles are their own

antiparticles. Hence, some phases of V_l^M can be absorbed only by redefining the charged lepton fields. The number of physical phase angles left in U^M is n(n+1)/2-n= n(n-1)/2. Thus U^M can be parameterized in terms of n(n-1)/2 rotation angles and the same number of phase angles. For the case n=3, we obtain the lepton mixing matrix with 3 rotation angles and 3 CP-violating phases.
Any 3×3 unitary matrix contains 3 moduli and 6 phases and can be written as

$$U=e^{i\varphi}P\widetilde{U_\nu}Q$$

where P=diag(1, $e^{i\varphi}, e^{i\omega}$) and Q=diag(1, $e^{i\rho}, e^{i\sigma}$) are diagonal phase matrices having 2 phases each, and $\widetilde{U_\nu}$ is dirac unitary matrix.
Following Particle Data Group (PDG) representation, wherein the unitarity is built in, involving three triangles θ_{12},θ_{23},θ_{13} and the Dirac like CP violating phase δ_l as well as two Majorana phases α_1, α_2, the PMNS matrix U can be written as:

$$U= U_l^{\dagger} U_\nu = e^{i\varphi}\widetilde{U_l^{\dagger}}\, P_\nu \widetilde{U_\nu} Q_\nu$$

$$=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_l} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_l} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_l} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_l} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_l} & c_{23}c_{13} \end{pmatrix}\begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ for i, j = 1, 2, 3.
In the scheme with three massive Majorana neutrinos under discussion there exist three rephasing invariants related to three CP violating phases in U, δ and $\alpha_{1,2}$.
The first is the standard Dirac one J_{CP} associated with Dirac phase δ

$$J_{CP}= Im(U_{e1}U_{\mu 1}U_{e2}^{*}U_{\mu 1}^{*}) = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin\delta_l ,$$

Where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and δ_l is the Dirac-like CP violating phase. It determines the magnitude of CP violation effects in neutrino oscillations. Let us note that if $U_l = 1$ and $\widetilde{U_\nu}$ is areal matrix, one has $J_{CP} = 0$.
The two additional invariants $S_1$and S_2, whose existence is related to the Majorana nature of massive neutrinos,S_1= $Im(U_{e1}U_{e3}^*)$,S_2= $Im(U_{e2}U_{e3}^*)$. If $S_1 \neq 0$ and $S_2 \neq 0$,CP is not conserved due to Majorana phases$\alpha_{1,2}$.The effective Majorana mass in $\beta\beta_{0\nu}$-decay $|M_{ee}|$,depends ,in general, on S_1 and S_2 and not on J_{CP}. Let us note, however even if,$S_{1,2}$(which can take place if e.g., $|U_{e3}|$=0),the two Majorana phases $\alpha_{1,2}$ can still be source of CP non conservation in the leptonic sector provided $Im(U_{e1}U_{e3}^*) \neq 0$ and $Im(U_{e2}U_{e3}^*) \neq 0$.

DIRAC VS MAJORANA NEUTRINOS

DIRAC MASS : Consider a fermion mass tem in the Lagrangian corresponds to

$$m_f \overline{\varphi_{R(L)}}\varphi_{L(R)} = \bar{f}_{L(R)} \rightarrow\leftarrow f_{L(R)}$$

Note that in the standard model a left handed or right handed) fermion–antifermion pair carries total isospin1/2,hence breaks gauge invariance of the lagrangian. So,even the quarks and charged leptons cannot have mass, represented by the above mass term in the lagrangian. Instead they get mass via their Yukawa coupling to isospin doublet of higgs boson, which acquires a vacuum expectation value by spontaneous breaking of isospin gauge symmetry i.e

$$y\overline{\varphi_{R(L)}}\varphi_{L(R)}h \xrightarrow{\text{sp. symm.breaking}} y\langle h\rangle\overline{\varphi_R}\varphi_L$$

In other words, their mass comes from their Yukawa interaction with the constant Higgs field <h>,present in the vaccum. This is called Dirac mass of quarks and charged leptons, which is roughly in the range of $\sim 10^{\pm 2}$ GeV.

MAJORANA MASS: We know that there is no right handed neutrino or left handed antineutrinos in the standard model, there cannot be any neutrino mass by combinations (except via Higgs Mechanism). But one can extend the standard model by introducing an isospin singlet $\nu_R(\nu_L)$ like other fermions it doesnot lead to symmetry of this model. Unlike the other fermions,however, the singlet $\nu_R(\nu_L)$ has a unique property i.e it doesnot carry any gauge charge. Although,, total lepton number is not conserved, it is not a gauge quantum number and hence not required to be conserved, hence such particles can get a new kind of mass called Majorana mass.

SEESAW MECHANISM:

Consider the Dirac and Majorana mass term,

$$L = m_D(\bar{\varphi}_L\varphi_R + \bar{\varphi}_R\varphi_L)$$

$$L_M = \frac{1}{2}m_L(\bar{\varphi}_L\varphi_R^c + \bar{\varphi}_R^c\varphi_L) + \frac{1}{2}m_R(\bar{\varphi}_R^c\varphi_R + \bar{\varphi}_R\varphi_L^c)$$

The most general case is ,in fact the combination of Dirac and Majorana picture. Therefore ,the Dirac-Majorana Lagrangian with mass matrix is

$$L_{MD} = (\varphi_L \bar{\varphi}_R^c)\begin{pmatrix} m_L^M & m^D \\ m^D & m_R^M \end{pmatrix}\begin{pmatrix} \varphi_R^c \\ \varphi_R \end{pmatrix}$$

where $m^M$and m^D are 3×3 matrices including flavor mixing.On introducing two independent Majorana fields, $\nu_1=\varphi_L + \varphi_R^c$ and $\nu_2=\varphi_R + \varphi_L^c$, we have

$$L_{DM} = \frac{1}{2}(\bar{\nu}_L \bar{\nu}_L^c).M.\begin{pmatrix} \nu_R^C \\ \nu_R \end{pmatrix}$$

Consider the special case of see-saw mechanism. Assuming that the scale of m_R is very large (around GUT scale) and the left handed majorana neutrino mass m_L is zero (in order to save the gauge invariance).Therefore, the Majorana mass becomes, $M=\begin{pmatrix} 0 & m_D \\ m_D & m_R^M \end{pmatrix}$

It has been observed that the heavier neutrino mass eigenvalues found to be of the order of GUT scale i.e $m_1 \sim 10^{15}$GeVand lighter neutrino mass eigenvalues comes out to be, $m_2 \approx -(m_D^2)/\ m_R^M \rightarrow 100^2\text{GeV}^2/10^{15}\text{GeV} \approx 0.01\text{eV}$.Thus,it explains why neutrino has smaller mass as compared to the charged leptons.

There is great deal of current interest in pursuing this model for two reasons: (1)In GUT,the lepton number L can be a gauge charge.In that case Majorana mass M will represent the GUT symmetry breaking scale.(2)The lepton no. violation,associated with Majorana mass scale, can generate a lepton asymmetry in the early universe, which will be able to explain the present baryon asymmetry in early universe.

TEXTURE SPECIFIC MASS MATRICES

The neutrino mass matrix contains all information about neutrino masses and mixings. However, the present neutrino data is not sufficient to determine the neutrino mass matrix. Therefore, we need some additional theoretical input to theoretical input to constraint the neutrino mass matrix. One of the most attractive ideas to constraint the neutrino mass matrix is to introduce certain number of zeros in the mass matrix. These zeros needn't be algebraically zeros but they can be phenomenological zeros by which we mean that the elements are not exactly zeros but many order of magnitudes smaller than the other non zero elements.

We can make different combinations of zeros by setting many elements of the neutrino mass matrix equal to zero. However, it was found that the presence of more than 2 zeros in matrix is unable to reproduce the current neutrino masses and mixing data. So, we 'll be discussing only two texture zeros.

There has been a long standing interest in the texture zeros of the 3×3 quark mass matrices as a possible source of the observed hierarchies in their masses and mixing angles. Frampton,Glashow andMarfatia have systematically compared the predictions of all the symmetric 3×3 neutrino mass matrices with two or more independent texture zeros with the neutrino mass and mixing parameters as derived from the oscillation data. Moreover, they find that 7 out of the 15 independent neutrino mass matrices with texture zeros ($^6C_2$=15) are compatible with the oscillation data.

EXPERIMENTAL CONSTRAINT

We shall impose the following constraints on the neutrino masses and mixing angles.

1. The CHOOZ and Paoloverde atomic experiments give the 90% CL limit[12]
   $\sin^2\theta_{13} < 0.041$
   The ratio of solar and atmospheric neutrino mass differences is
   $R_\nu = \left|\frac{m_2^2-m_1^2}{m_3^2-m_{1,2}^2}\right| = \Delta m^2_{sun}/\Delta m^2_{atm} << 2\times10^{-2}$
2. The effective mass term of neutrino less double beta decay is
   $$|M_{ee}| = m_3\left|\frac{m_1}{m_2}U_{e1}^2 e^{2i\rho} + \frac{m_2}{m_3}U_{e2}^2 e^{2i\sigma} + U_{e3}^2\right|$$

The Heidelberg-Moscow Collaboration has reported $|M_{ee}| < 0.34$eV at 90% CL[13]

NEUTRINO MASS MATRICES WITH TWO ZERO TEXTURE

In the flavor basis where the charged lepton mass matrix is diagonal, the neutrino mass matrix can be written as

$$M = V\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}V^T$$

where $m_i(1,2,3)$denote the neutrino masses and V is lepton flavor mixing matrix.

A full description of V needs six real parameters: three mixing angles and three CP-violating phases. Note that V can always be expressed as a product of Dirac –type flavor mixing matrix U(consisting of three mixing angles & three CP- violating phase) and diagonal phase matrix P (consisting of two non-trivial Majorana phases):V=UP.

$$M = U\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}U^T$$

Use following parameterization for Dirac–type flavor mixing matrix

$$U = \begin{pmatrix} c_x c_z & s_x c_z & s_z \\ -c_x s_y s_z - s_x c_y e^{i\delta} & -s_x s_y s_z + c_x c_y e^{-i\delta} & s_y c_z \\ -c_x c_y s_z + s_y s_x e^{-i\delta} & -s_x c_y s_z - c_x s_y e^{-i\delta} & c_y c_z \end{pmatrix}$$

where $s_x \equiv \sin\theta_x$, $c_x \equiv \cos\theta_x$ and $\lambda_1 = m_1 e^{2i\rho}, \lambda_2 = m_2 e^{2i\sigma}, \lambda_3 = m_3$ *are Majorana phases*

As M is Symmetric, it has six independent entries, if two of them vanishes,

i.e $M_{ab}=M_{\alpha\beta} = 0$,we obtain constraint equation

$$\sum_{i=1}^{3} U_{ai}U_{bi}\lambda_i = 0, \qquad \sum_{i=1}^{3} U_{\alpha i}U_{\beta i}\lambda_i = 0$$

Using above, we get

$$\frac{\lambda_1}{\lambda_3} = \frac{U_{a3}U_{b3}U_{\alpha 2}U_{\beta 2} - U_{a2}U_{b2}U_{\alpha 3}U_{\beta 3}}{U_{a2}U_{b2}U_{\alpha 1}U_{\beta 1} - U_{a1}U_{b2}U_{\alpha 2}U_{\beta 2}}, \qquad \frac{\lambda_2}{\lambda_3} = \frac{U_{a1}U_{b1}U_{\alpha 3}U_{\beta 3} - U_{a3}U_{b3}U_{\alpha 1}U_{\beta 1}}{U_{a2}U_{b2}U_{\alpha 1}U_{\beta 1} - U_{a1}U_{b2}U_{\alpha 2}U_{\beta 2}}$$

Also, the neutrino mass ratios,we find

$$\frac{m_1}{m_2} = \left|\frac{U_{a3}U_{b3}U_{\alpha 2}U_{\beta 2} - U_{a2}U_{b2}U_{\alpha 3}U_{\beta 3}}{U_{a2}U_{b2}U_{\alpha 1}U_{\beta 1} - U_{a1}U_{b2}U_{\alpha 2}U_{\beta 2}}\right|,$$

$$\frac{m_2}{m_3} = \left|\frac{U_{a1}U_{b1}U_{\alpha 3}U_{\beta 3} - U_{a3}U_{b3}U_{\alpha 1}U_{\beta 1}}{U_{a2}U_{b2}U_{\alpha 1}U_{\beta 1} - U_{a1}U_{b2}U_{\alpha 2}U_{\beta 2}}\right|$$

The expression for two majorana phases are found to be

$$\rho = \frac{1}{2} arg \left|\frac{U_{a3}U_{b3}U_{\alpha 2}U_{\beta 2} - U_{a2}U_{b2}U_{\alpha 3}U_{\beta 3}}{U_{a2}U_{b2}U_{\alpha 1}U_{\beta 1} - U_{a1}U_{b2}U_{\alpha 2}U_{\beta 2}}\right|$$

$$\sigma = \frac{1}{2} arg \left|\frac{U_{a1}U_{b1}U_{\alpha 3}U_{\beta 3} - U_{a3}U_{b3}U_{\alpha 1}U_{\beta 1}}{U_{a2}U_{b2}U_{\alpha 1}U_{\beta 1} - U_{a1}U_{b2}U_{\alpha 2}U_{\beta 2}}\right|$$

(A) HIERARCHICAL SOLUTION

Pattern A_1: $(i.e\ a = b = e, \alpha = e\ \&\ \beta = \mu)$ we obtain

$$\frac{\lambda_1}{\lambda_3} = \frac{s_z}{c_z^2}\left(\frac{s_x s_y}{c_x c_y} e^{i\delta} - s_z\right)$$

$$\frac{\lambda_2}{\lambda_3} = -\frac{s_z}{c_z^2}\left(\frac{c_x s_y}{s_x c_y} e^{i\delta} - s_z\right)$$

For input $\sin^2 2\theta_x \sim 0(1), \sin^2 2\theta_y \sim 1\ \&\ \sin^2 2\theta_z \leq 0.1$,[14,15]

We have $\frac{m_1}{m_3} = t_x t_y s_z, \frac{m_2}{m_3} \approx \frac{t_y}{t_x} s_z$; $\rho \approx \frac{\delta}{2}, \sigma \approx \frac{\delta}{2} \pm \frac{\pi}{2}$;

$$R_\nu \approx \frac{t_y^2}{t_x^2}|1 - t_x^4|s_z^2; |M_{ee}| = 0$$

Taking typical inputs $\theta_x = 30^o, \theta_y = 40^o, \theta_z = 5^o\ \&\ \delta = 90^o$,we obtain $\frac{m_1}{m_3} = 0.04, \frac{m_2}{m_3} = 0.13, \rho = 45^o\ \&\sigma \sim 135^o, R_\nu = .014$ The value of R_ν shows that solar neutrino deficit is attributed to large angle MSW oscillation and $|M_{ee}| \sim 0$, implies it is impossible to detect $0\nu\beta\beta$ decay.

Pattern A_2: $M_{ee} = M_{e\tau} = 0 (i.e\ a = b = e, \alpha = e\ \&\ \beta = \tau)$

$$\frac{\lambda_1}{\lambda_3} = -\frac{s_z}{c_z^2}\left(\frac{s_x c_y}{c_x s_y}e^{i\delta} + s_z\right)$$

$$\frac{\lambda_2}{\lambda_3} = \frac{s_z}{c_z^2}\left(\frac{c_x c_y}{s_x s_y}e^{i\delta} - s_z\right)$$

In lowest order approximation, $\frac{m_1}{m_3} = \frac{t_x}{t_y}s_z, \frac{m_2}{m_3} \approx \frac{1}{t_x t_y}s_z\ ; \sigma \approx \frac{\delta}{2}, \rho \approx \frac{\delta}{2} \pm \frac{\pi}{2};$

$$R_v \approx \frac{1}{t_x^2 t_y^2}|1 - t_x^4|s_z^2; |M_{ee}| = 0$$

Using same inputs as above,we obtain $\frac{m_1}{m_3} = 0.06, \frac{m_2}{m_3} = 0.18, \rho = 135^o$ and $\sigma \sim 45^o, R_v = .03$ same as A_1

Therefore, we have $|m_1| < |m_2| \ll |m_3|$, i.e. hierarchical masses.

(B) DEGENERATE SOLUTION

PatternB$_1$: $M_{\mu\mu} = M_{e\tau} = 0 (i.e\ a = b = e, \alpha = e\ \&\ \beta = \tau)$,

we have

$$\frac{\lambda_1}{\lambda_3} = \frac{s_x c_x s_y\left(2c_y^2 s_z^2 - s_y^2 c_z^2\right) - c_y s_z\left(s_x^2 s_y^2 e^{i\delta} + c_x^2 c_y^2 e^{-i\delta}\right)}{s_x c_x s_y c_y^2 + (s_x^2 - c_x^2)c_y^3 s_z e^{i\delta} + s_x c_x s_y s_z^2(1 + c_y^2)e^{2i\delta}}\, e^{2i\delta}$$

$$\frac{\lambda_2}{\lambda_3} = \frac{s_x c_x s_y\left(2c_y^2 s_z^2 - s_y^2 c_z^2\right) + c_y s_z\left(c_x^2 s_y^2 e^{i\delta} + s_x^2 c_y^2 e^{-i\delta}\right)}{s_x c_x s_y c_y^2 + (s_x^2 - c_x^2)c_y^3 s_z e^{i\delta} + s_x c_x s_y s_z^2(1 + c_y^2)e^{2i\delta}}\, e^{2i\delta}$$

To lowest order, we have $\frac{m_1}{m_3} = \frac{m_2}{m_3} \approx t_y^2\ ; \rho \approx \sigma \approx \frac{\delta}{2} \pm \frac{\pi}{2};$

$$R_v \approx \frac{1 + t_x^2}{t_x}\left|t_{2y} c_\delta\right|; |M_{ee}| \approx m_3 t_y^2$$

Taking typical inputs $\theta_x = 30^o, \theta_y = 40^o, \theta_z = 5^o\ \&\ \delta = 89^o$

$\frac{m_1}{m_3} = \frac{m_2}{m_3} \approx 0.7\ ; \rho \approx \sigma \approx 179^o$ (difference of about 3^o), $R_v \approx .002\ and\ \frac{|M_{ee}|}{m_3} \approx 0.7$

If B_1 is correct, large CP violation is observable in LBL. Also, we observe that B_1 is nearly degenerate, $m_1 \sim m_2 \sim m_3$. Heidelberg-Moscow experiment gives upper bound on $|M_{ee}| < 0.34eV$, compatible with present Direct mass–search experiment.

PatternB$_2$: $M_{\tau\tau} = M_{e\mu} = 0 (i.e\ a = b = \tau, \alpha = e\ \&\ \beta = \mu)$

$$\frac{\lambda_1}{\lambda_3} = \frac{s_x c_x c_y\left(2s_y^2 s_z^2 - s_y^2 c_z^2\right) + s_y s_z\left(s_x^2 c_y^2 e^{i\delta} + c_x^2 s_y^2 e^{-i\delta}\right)}{s_x c_x c_y s_y^2 - (s_x^2 - c_x^2)s_y^3 s_z e^{i\delta} + s_x c_x c_y s_z^2(1 + s_y^2)e^{2i\delta}}\, e^{2i\delta}$$

$$\frac{\lambda_2}{\lambda_3} = \frac{s_x c_x s_y\left(2s_y^2 s_z^2 - c_y^2 c_z^2\right) - s_y s_z\left(c_x^2 c_y^2 e^{i\delta} + s_x^2 s_y^2 e^{-i\delta}\right)}{s_x c_x c_y s_y^2 - (s_x^2 - c_x^2)s_y^3 s_z e^{i\delta} + s_x c_x s_y s_z^2(1 + s_y^2)e^{2i\delta}}\, e^{2i\delta}$$

we obtain, $\frac{m_1}{m_3} = \frac{m_2}{m_3} \approx \frac{1}{t_y^2}$; $\rho \approx \sigma \approx \frac{\delta}{2} \mp \frac{\pi}{2}$;

$$R_\nu \approx \frac{1+t_x^2}{t_x}\left|t_{2y}c_\delta\right|; |M_{ee}| \approx m_3/t_y^2$$

<u>PatternB3:</u> $M_{\mu\mu} = M_{e\mu} = 0(i.e\ a = b = \mu, \alpha = e\ \&\ \beta = \mu)$

$$\frac{\lambda_1}{\lambda_3} = -\frac{s_y}{c_y}\frac{s_x s_y - c_x c_y s_z e^{-i\delta}}{s_x c_y + c_x s_y s_z e^{i\delta}} e^{2i\delta}$$

$$\frac{\lambda_2}{\lambda_3} = -\frac{s_y}{c_y}\frac{c_x s_y + s_x c_y s_z e^{-i\delta}}{c_x c_y - s_x s_y s_z e^{i\delta}} e^{2i\delta}$$

We have $\frac{m_1}{m_3} = \frac{m_2}{m_3} \approx t_y^2$; $\rho \approx \sigma \approx \frac{\delta}{2} \pm \frac{\pi}{2}$;

$$R_\nu \approx \frac{1+t_x^2}{t_x}\left|t_{2y}c_\delta\right|; |M_{ee}| \approx m_3 t_y^2$$

<u>Pattern B4:</u> $M_{\tau\tau} = M_{e\tau} = 0(i.e\ a = b = \tau, \alpha = e\ \&\ \beta = \tau)$

$$\frac{\lambda_1}{\lambda_3} = -\frac{c_y}{s_y}\frac{s_x s_y + c_x c_y s_z e^{-i\delta}}{s_x s_y - c_x c_y s_z e^{i\delta}} e^{2i\delta} \quad \frac{\lambda_2}{\lambda_3} = -\frac{c_y}{s_y}\frac{c_x c_y - s_x s_y s_z e^{-i\delta}}{s_x c_y + c_x s_y s_z e^{i\delta}} e^{2i\delta}$$

For lowest order approximation ,we obtain $\frac{m_1}{m_3} = \frac{m_2}{m_3} \approx \frac{1}{t_y^2}$; $\rho \approx \sigma \approx \frac{\delta}{2} \pm \frac{\pi}{2}$;

$$R_\nu \approx \frac{1+t_x^2}{t_x}\left|t_{2y}c_\delta\right|; |M_{ee}| \approx m_3/t_y^2$$

One conclude that using the same input as in B_1,the phenomenological consequences of pattern B_1 ,B_2 ,$B_3$and B_4 are same

<u>Pattern: C</u> $M_{\mu\mu} = M_{\tau\tau} = 0(i.e\ a = b = \mu, \alpha = \beta = \tau)$

$$\frac{\lambda_1}{\lambda_3} = -\frac{c_x c_z^2}{s_z}\frac{c_x(s_y^2 - c_y^2) + 2s_x s_y c_y s_z e^{i\delta}}{2s_x c_x s_y c_y - (s_x^2 - c_x^2)(s_y^2 - c_y^2)s_z e^{i\delta} + 2s_x c_x s_y c_y s_z^2 e^{2i\delta}} e^{i\delta}$$

$$\frac{\lambda_2}{\lambda_3} = \frac{s_x c_z^2}{s_z}\frac{s_x(s_y^2 - c_y^2) - 2c_x s_y c_y s_z e^{i\delta}}{2s_x c_x s_y c_y - (s_x^2 - c_x^2)(s_y^2 - c_y^2)s_z e^{i\delta} + 2s_x c_x s_y c_y s_z^2 e^{2i\delta}} e^{i\delta}$$

Assuming, $s_z^2 \ll 1\ \&t_x \sim t_y \sim 0(1)$, we have $\frac{m_1}{m_3} \approx \left(1 - \frac{2c_\delta}{t_x t_{2y} s_z} + \frac{1}{t_x^2 t_{2y}^2 s_z^2}\right)^{1/2}$, $\frac{m_2}{m_3} \approx \left(1 + \frac{2t_x c_\delta}{t_{2y} s_z} + \frac{t_x^2}{t_{2y}^2 s_z^2}\right)^{1/2}$

$$R_\nu \approx \frac{1+t_x t_y}{t_x t_y}\left|\frac{2}{t_{2x}} \cdot \frac{1 - t_{2x}t_{2y}s_z c_\delta}{t_x + 2c_\delta t_{2y} s_z}\right| \& |M_{ee}| \approx m_3\left(1 - \frac{4c_\delta}{t_{2x}t_{2y}s_z} + \frac{4}{t_{2x}t_{2y}s_z^2}\right)^{1/2}$$

Also, $\rho \approx \delta + \epsilon \pm \frac{\pi}{2}$ with $t_\epsilon = \frac{s_\delta}{t_x t_{2y} s_z - c_\delta}$ & $\sigma \approx \delta + \epsilon \pm \frac{\pi}{2}$ with $t_\epsilon = \frac{t_x s_\delta}{t_{2y} s_z + t_x c_\delta}$

Taking input, $\theta_x = \theta_y = 44.8^o, \theta_z = 5^o$ & $\delta = 90^o$, we find $R_\nu \approx .03, \rho \approx +5^o \sigma \approx -5^o$. Like Category B, C also have nearly degenerate neutrino mass spectrum. One can also observe that can have R_ν small finite value iff $t_{2x} t_{2y} s_z c_\delta \approx 1$ is satisfied [16].

Let us summarize the main phenomenological consequences of each category:

The Dirac like CP violating phase(δ) is not constrained in category A; $\delta \approx \frac{\pi}{2}$ in B and is not sensitive to the values of mixing angles in pattern C. On the other hand, the Majorana phases are $|\sigma - \rho| \approx \frac{\pi}{2}$ *in category A*, $\sigma \approx \rho$ in category B; *and C*. Moreover, the neutrino less double $\beta decay$: $|M_{ee}| \approx 0$ *in category A*, $|M_{ee}| \approx m_3$ in category B &C

The remaining 8 mass matrices are experimentally disallowed as they are not compatible with experimental constraints as discussed above.

Pattern	Constraint Condition	Texture of M	Order of Magnitude
A_1	$M_{ee} = M_{e\mu} = 0$	$\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$	$\sim m_3 \begin{pmatrix} 0 & 0 & .1 \\ 0 & .4 & .5 \\ .1 & .5 & .6 \end{pmatrix}$
A_2	$M_{ee} = M_{e\tau} = 0$	$\begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}$	$\sim m_3 \begin{pmatrix} 0 & .1 & 0 \\ .1 & .4 & .5 \\ 0 & .5 & .6 \end{pmatrix}$
B_1	$M_{\mu\mu} = M_{e\tau} = 0$	$\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$	$\sim m_3 \begin{pmatrix} .7 & .06 & 0 \\ .06 & 0 & .8 \\ 0 & .8 & .3 \end{pmatrix}$
B_2	$M_{\tau\tau} = M_{e\mu} = 0$	$\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$	$\sim m_1 \begin{pmatrix} 1 & 0 & .05 \\ 0 & .3 & .8 \\ .05 & .8 & 0 \end{pmatrix}$
B_3	$M_{\mu\mu} = M_{e\mu} = 0$	$\begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$	$\sim m_3 \begin{pmatrix} 0 & 0 & .1 \\ 0 & .4 & .5 \\ .1 & .5 & .6 \end{pmatrix}$
B_4	$M_{\tau\tau} = M_{e\tau} = 0$	$\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}$	$\sim m_1 \begin{pmatrix} .1 & .04 & 0 \\ .04 & .3 & .8 \\ 0 & .8 & 0 \end{pmatrix}$
C	$M_{\mu\mu} = M_{\tau\tau} = 0$	$\begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$	$\sim m_3 \begin{pmatrix} 1 & .06 & .2 \\ .06 & 0 & 1 \\ .2 & 1 & 0 \end{pmatrix}$

SUMMARY AND CONCLUSION

Starting with neutrinos and its basic properties, we have discussed briefly about Dirac and Majorana masses of neutrinos. We explained how the neutrino get the Dirac mass from its Yukawa interaction with the constant Higgs field <h>, present in the vacuum. In addition, we have focused on the see-saw model and show how it leads to smallness of neutrino mass by simple calculations. Apart from brief discussion on the history of neutrino oscillation, we understand the theory behind neutrino oscillation and neutrino mixing. We discuss the case of neutrino oscillation in two flavors and three flavors.

In addition to this, experimental evidence for solar neutrino oscillation, atmospheric neutrino oscillation and reactor has been discussed in detail.

We discuss the seven allowed two-zero neutrino textures. They fall into three classes: A(with two members),B(with four members),and C. The textures within each class are difficult or impossible to distinguish experimentally, but each of the three classes has radically different implications.

It is surprising that such a great variety of textures of neutrino mass matrix can fit to what is presently known data about neutrino masses and oscillations. Future data should reveal which, if any of these textures should serve as a guide to model builder.

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